

EPFL Cours d'Electrotechnique I

2. Conventions et les symboles:

→ concepts → modifs

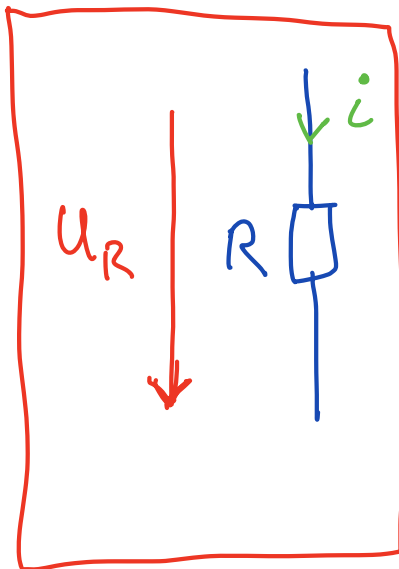
Ex : courant : i , I , $\hat{i}$, $\hat{I}$, $\bar{i}$, $\bar{I}$

unité : $[A]$

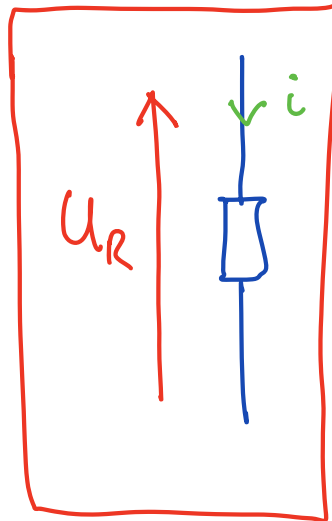
Relations : $u = R \cdot I$
 $u = R \cdot i$

Dessin : 
Résistance

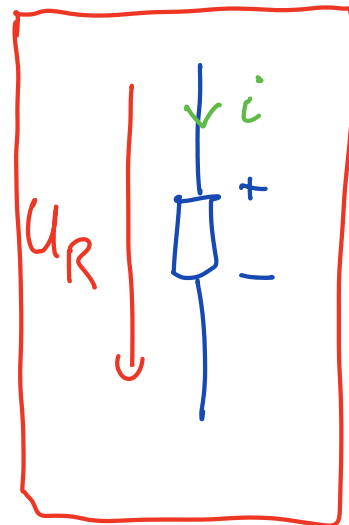
Choix à faire :



International



F_n



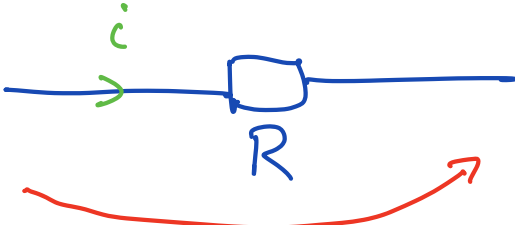
USA, B

Convention moteur : choix

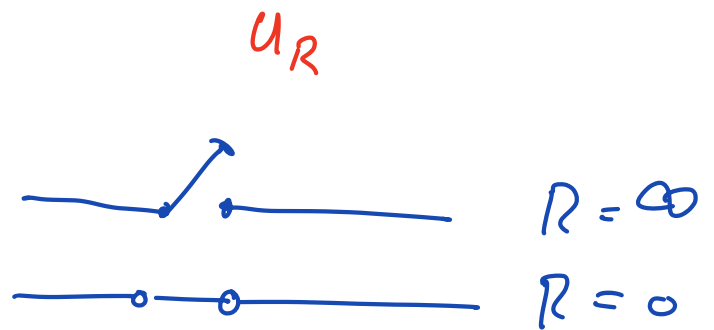
2.2 Représentation graphique :

Conducteur : 
parfait

Conducteur : 
avec un courant

Élément : 

Interrupteur :

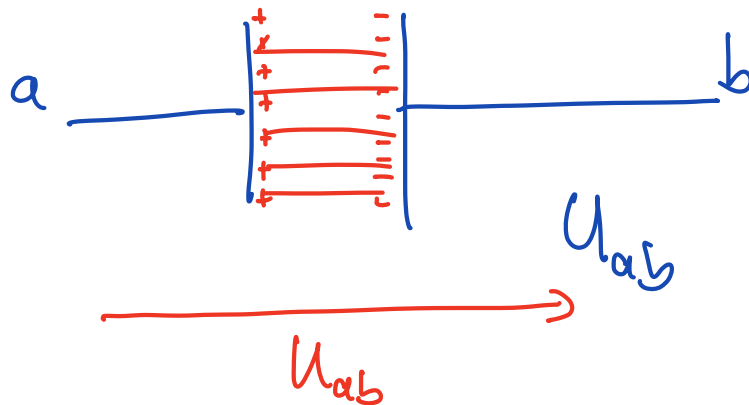


3. Lois fondamentales :

Différence de potentiel : Tension [Volt]

$$V_a - V_b = \int_a^b E dl = U_{ab} \quad [V]$$

$\xleftarrow{\quad l \quad} \xrightarrow{\quad}$



3.2.19 La Capacité :

Définition : Charge électrique : Q

Capacité :

$$C = \frac{Q}{U_{ab}}$$

Symbolle : 

3.3 Courant électrique :

$$I = \frac{dQ}{dt} \quad [A]$$

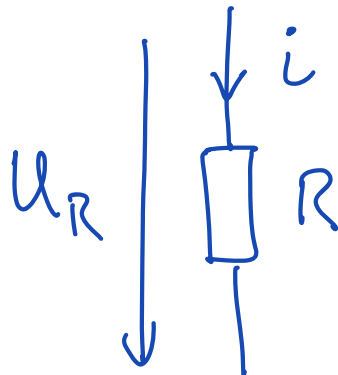
Densité de courant : $\mathbf{j} \Rightarrow [A/m^2]$

3.3.4 Pertes Joule :

$$P = R \cdot I^2 \quad [W]$$

Récap :

Convention Notion :



→ Puissance positive

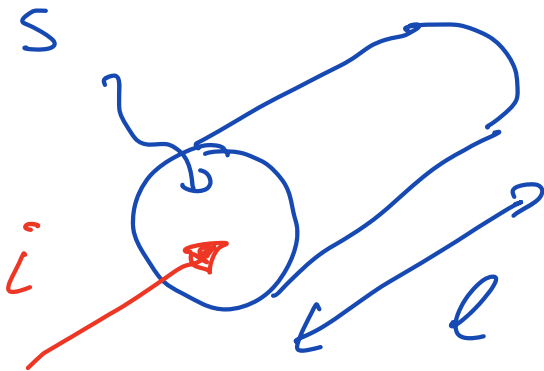
→ conv. Notion consommation.

3.3.6 Définition de la résistance :

$$R_{ab} = \int_a^b \rho \cdot \frac{dl}{S}$$

$\uparrow$ résistivité électrique $[\Omega m]$

$\leftarrow$ longueur
 $\nwarrow$ surface



Si S est constante sur la longueur

$$R_{ab} = \frac{\rho \cdot l}{S} \quad [\Omega]$$

3.3.8 Loi d'Ohm :

$$U_{ab} = R_{ab} I$$

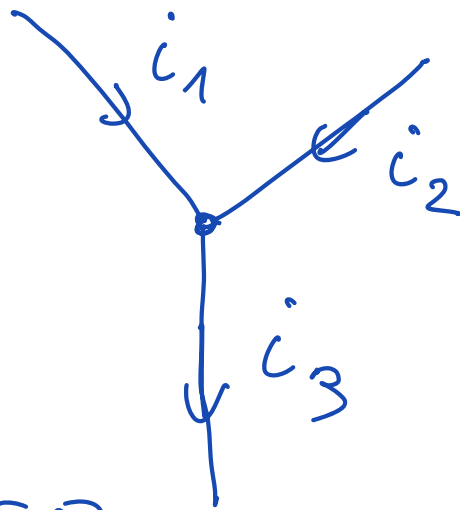
(courant et tension continue)

$$U_{ab} = R_{ab} \cdot i \quad (\text{courant et tension variable})$$

3.3.11 Lois de Kirchhoff :

Noeud : Point de convergence
d'au moins trois conducteurs

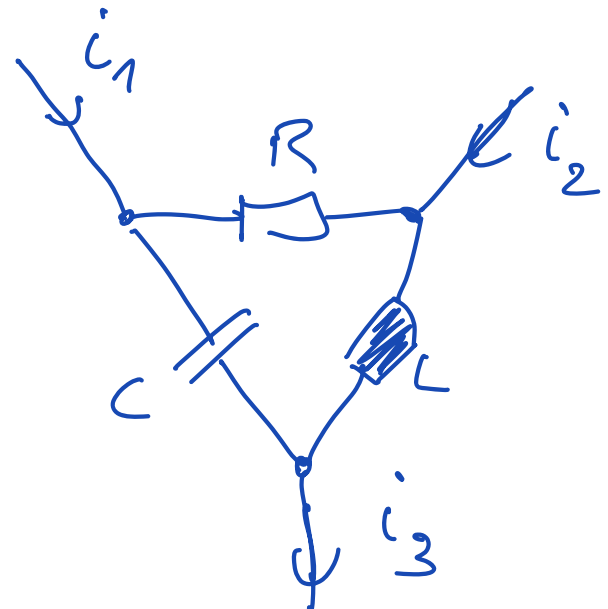
$$\underline{\sum i_j = 0}$$



$$i_1 + i_2 - i_3 = 0$$

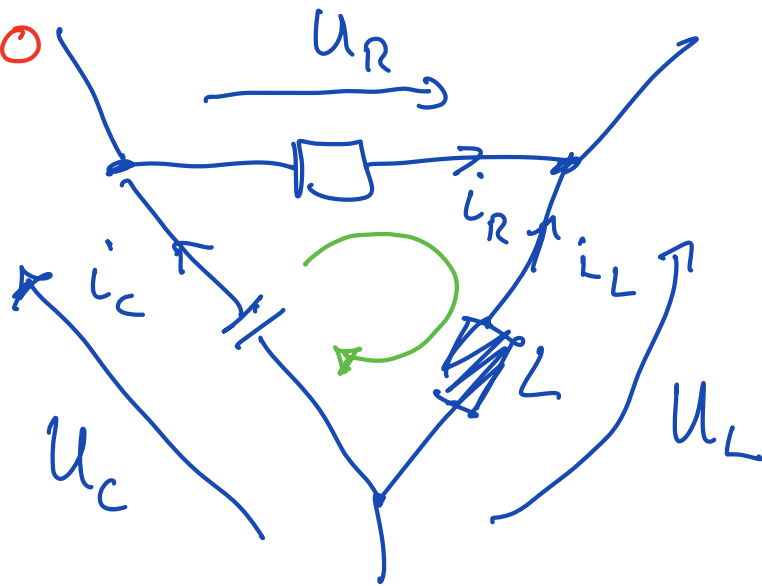
Noeud généralisé :

$$i_1 + i_2 - i_3 = 0$$



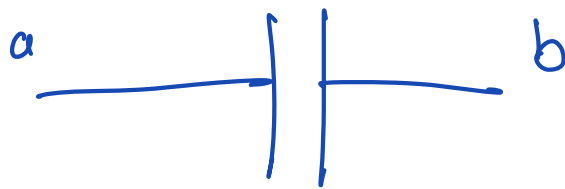
Maille : ensemble de branche partant d'un nœud pour y retourner

$$\sum U_j = 0$$



$$U_R - U_L + U_C = 0$$

3.5 La Capacité :

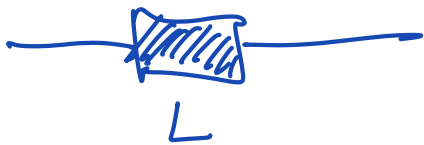


$$C = \frac{Q}{U_{ab}}$$

$$Q = \int i dt$$

$$u = \frac{1}{C} \int i dt$$

3.4 l'inductance :



$$u = L \frac{di}{dt}$$

$$\begin{cases} \vec{\text{Rot}} \vec{H} = \vec{J} \\ \vec{\text{Rot}} \vec{E} = -\frac{d\vec{B}}{dt} \end{cases}$$

$L [H]$ Henry

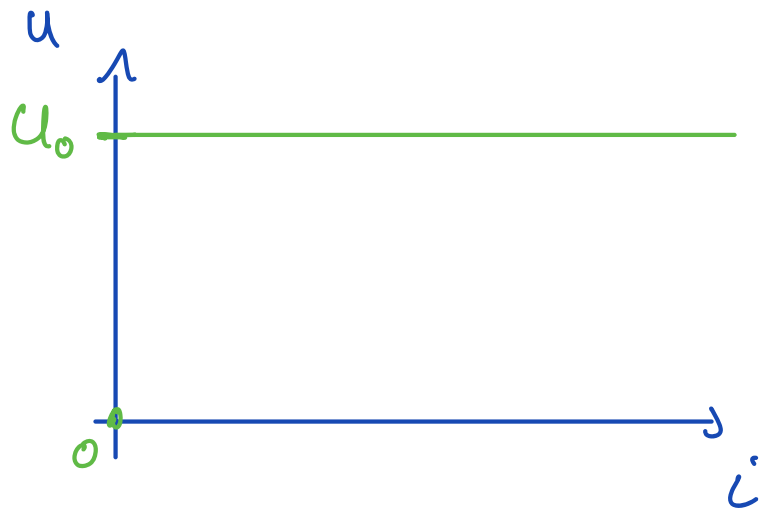
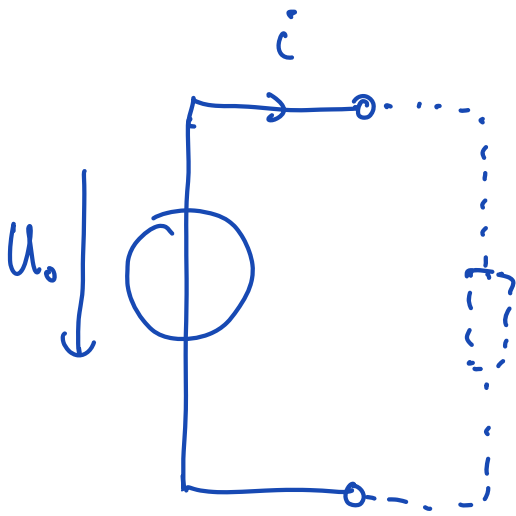
4. Eléments de circuit :

4.1 Dipôle : circuit qui possède 2 bornes



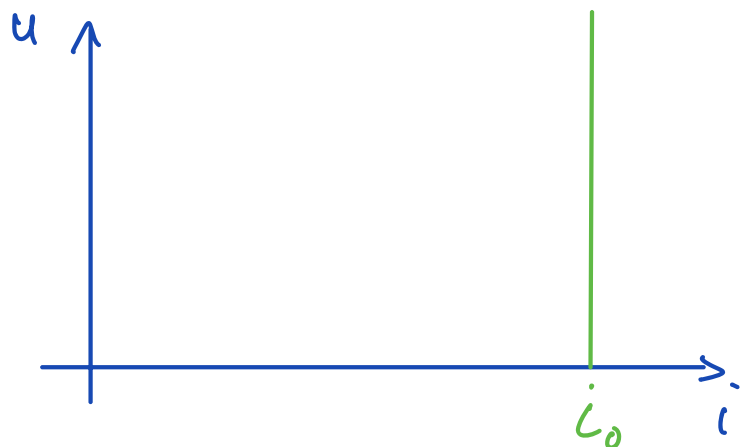
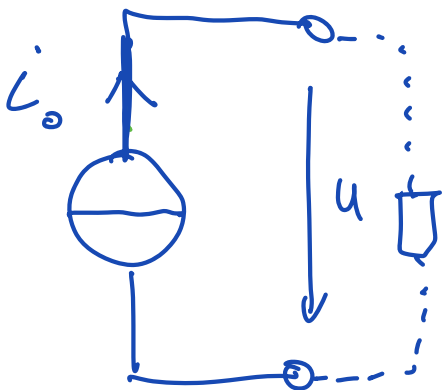
4.2 Sources de tension et de courant

a) Source de tension idéale :



c'est un élément virtuel, idéal et inexistant dans la nature

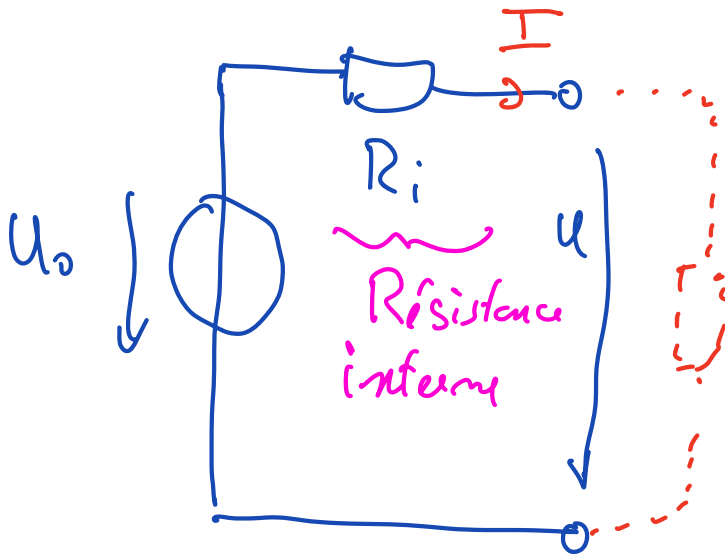
b) Source de courant idéale :



élément virtuel, inexistant dans la nature.

4.2.5 Source de tension réelle :

Def :



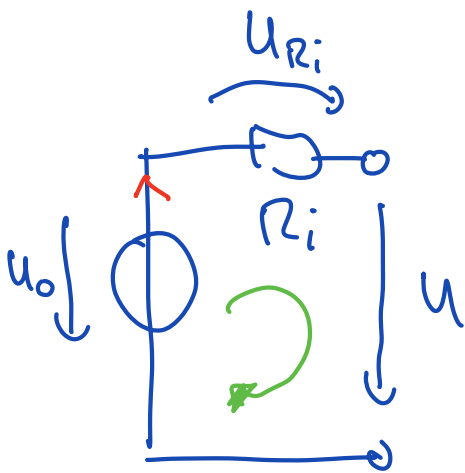
S. Tension
idéale

S. tension réelle

U_0 : Tension de
la source idéale
Tension à vide

R_i : Résistance
interne

U : Tension de
la source

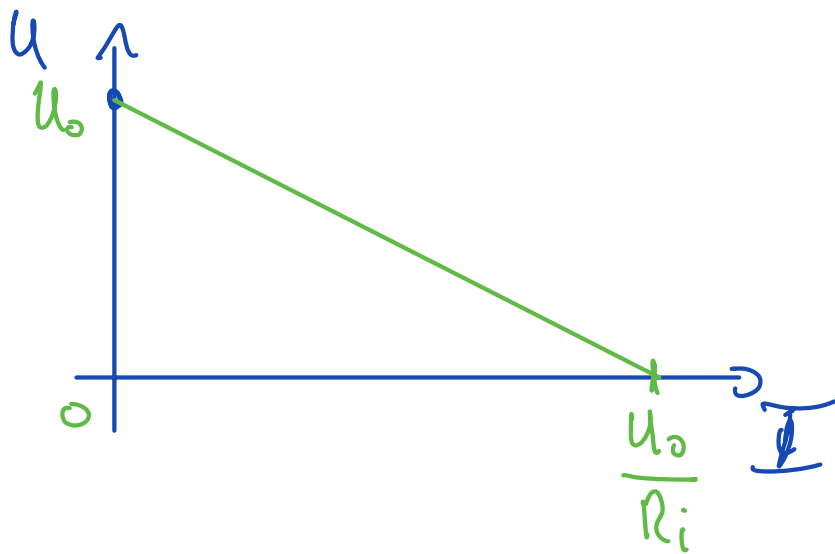


$$\sum U = 0$$

$$-U_0 + U_{R_i} + U = 0$$

$= R_i \cdot I$

$$U = U_0 - R_i \cdot I$$



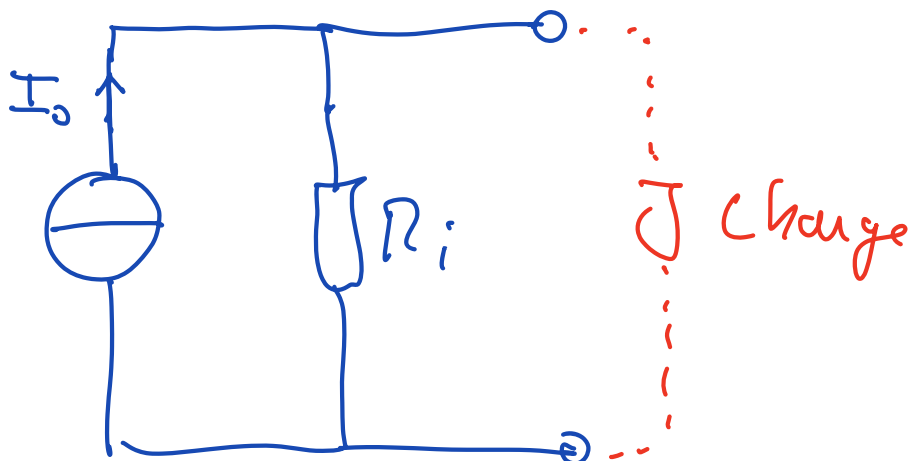
Courant I_{cc} :

$$U = 0$$

$$0 = U_0 - R_i \cdot I_{cc}$$

$$I_{cc} = \frac{U_0}{R_i}$$

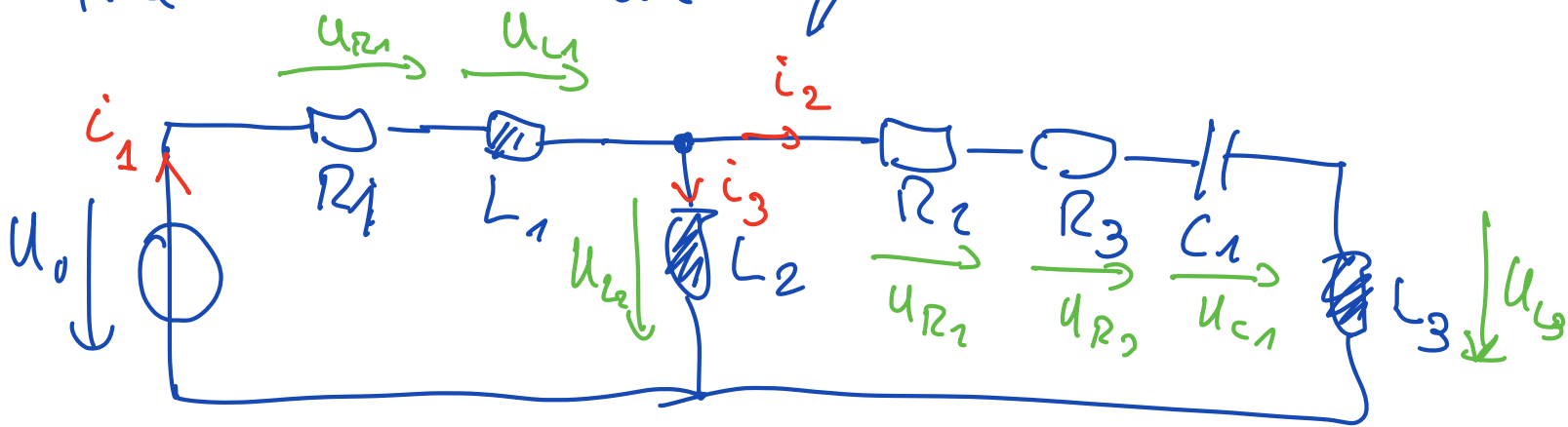
4.2.6 Source de courant réelle:



4.3 Elément de base:

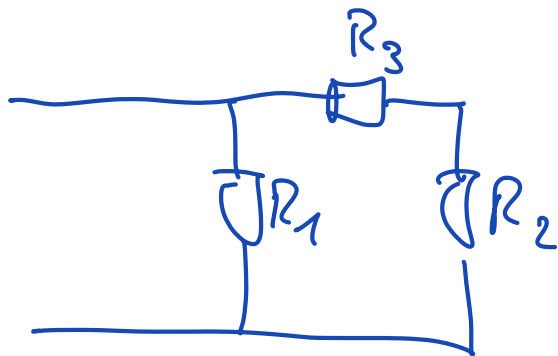


4.4 Scina electropu :



Recap: Quiz :

4: // $\rightarrow$ même tension aux bornes



R_1 n'est pas
en // avec R_2 !

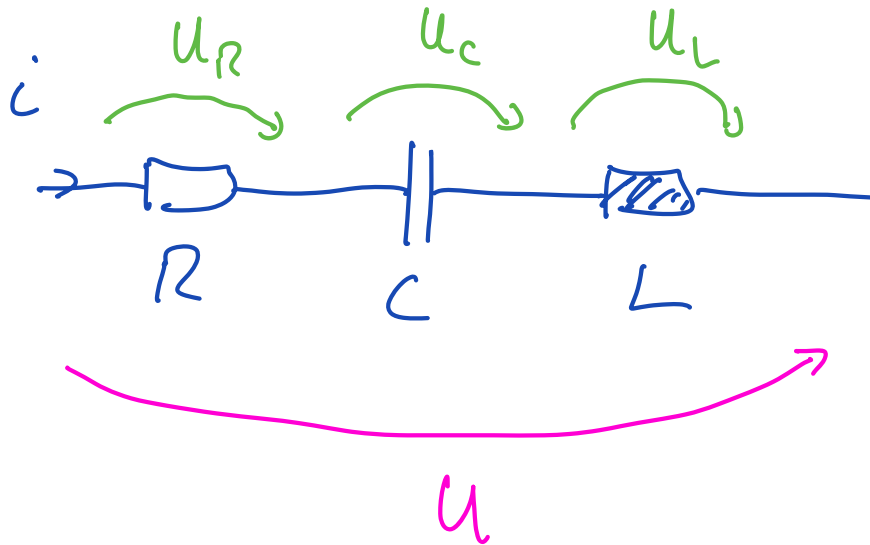
7: Source ideal sehr impossible.

8: Some ideal est toujours constante

q: Impossible de mettre des sautes de court en série.

5. Combinaison simple d'éléments linéaires

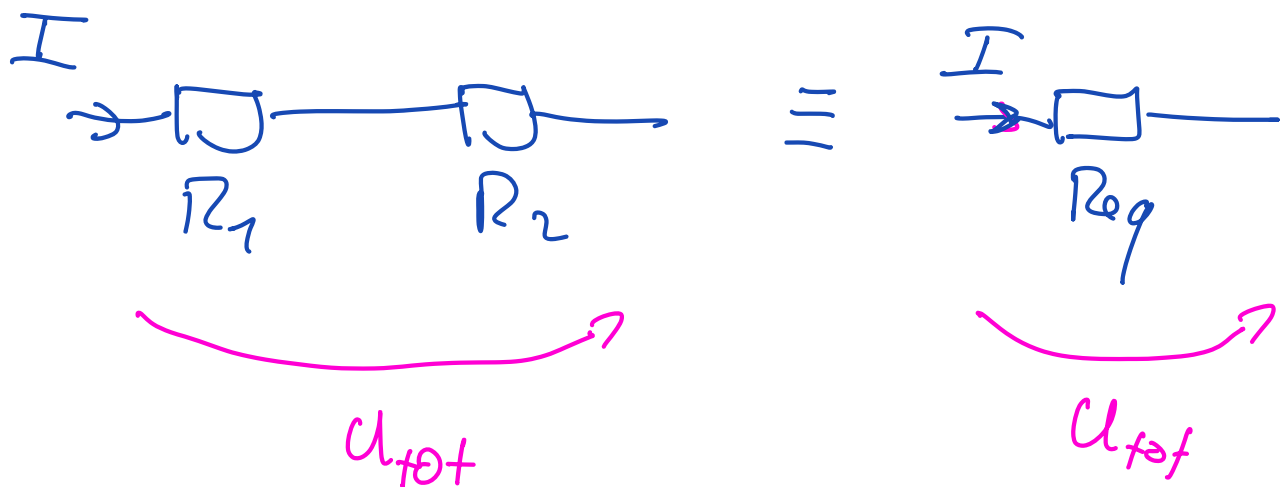
5.2 Mise en série :



Série : parcouru par le même courant

$$i_R = i_L = i_C \rightarrow \text{Série}$$

5.2.2 Mise en série de la résistance



$$U_{\text{tot}} = U_{R_1} + U_{R_2}$$

$$U_{\text{tot}} = R_{\text{eq}} \cdot I$$

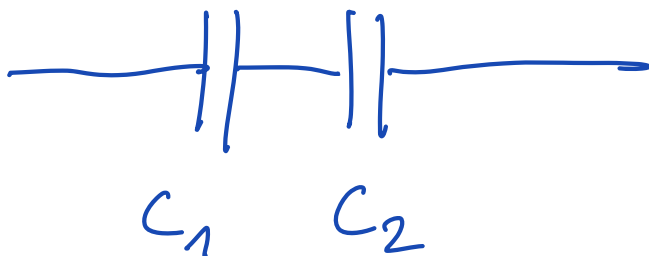
$$= R_1 I + R_2 I$$

$$= (R_1 + R_2) I = R_{\text{eq}} \cdot I$$

$$\Rightarrow R_{\text{eq}} = R_1 + R_2$$

En série $R_{\text{eq}} = \sum_{k=1}^m R_k$ ($m = \text{nb de } R$)

5.2.3 Mise en Série des C



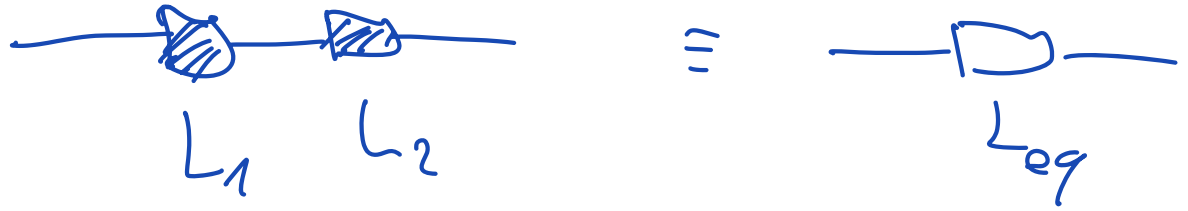
$$\equiv C_{\text{eq}} = ?$$



Série $C_{\text{eq}} = \frac{1}{\sum_{k=1}^m \frac{1}{C_k}}$

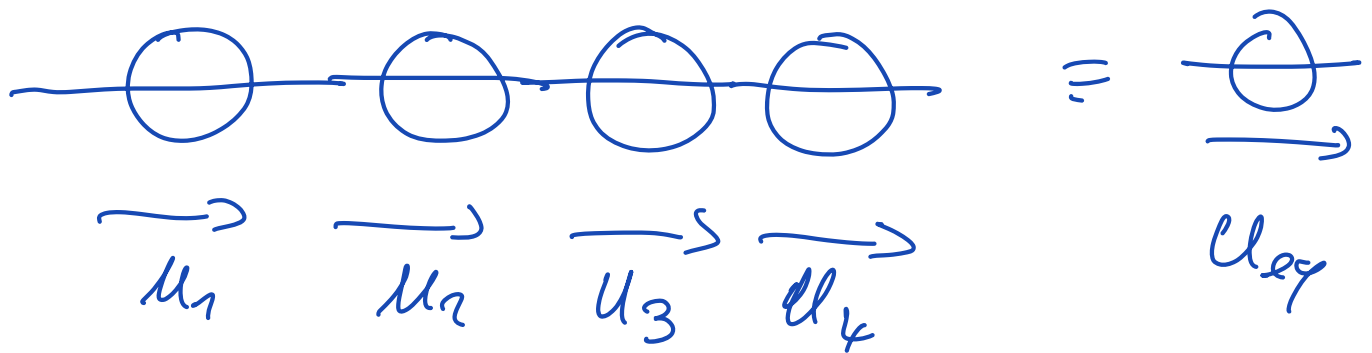
$m = \text{nb de } C$

5.2.6 Mise en série des L

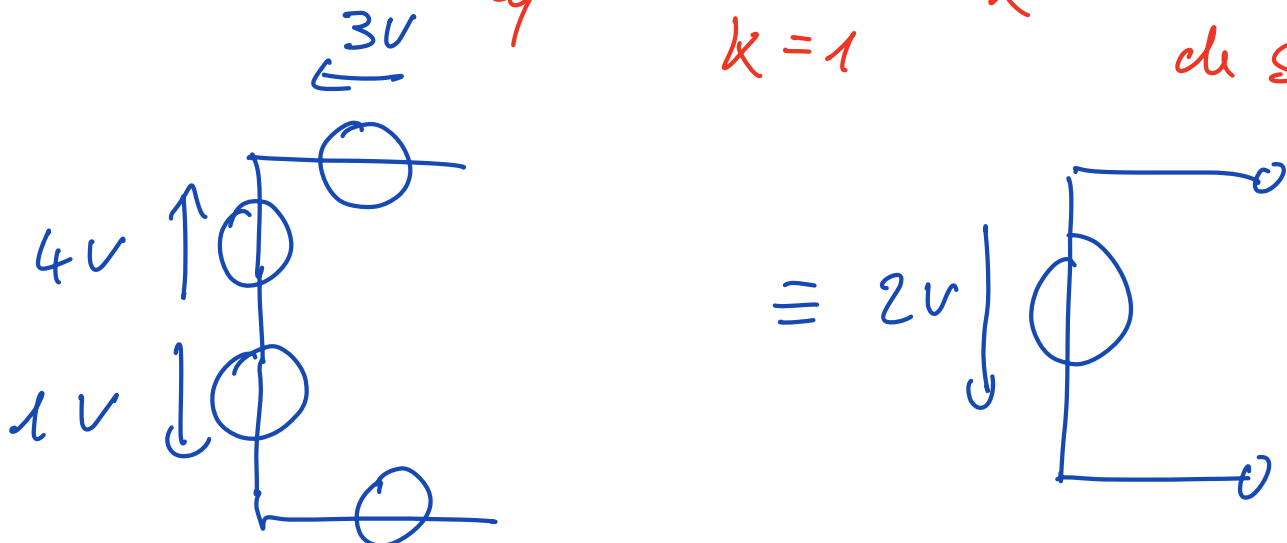


Série $L_{eq} = \sum_{k=1}^m L_k$ $m = nb \text{ de } L$

5.2.7 Mise en série de Source de tension

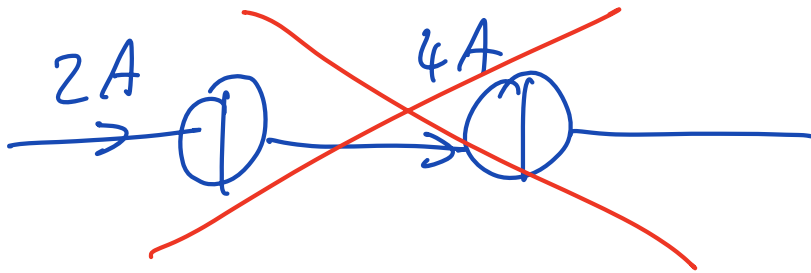


Série $U_{eq} = \sum_{k=1}^m U_k$ $m = nb \text{ de source}$



$\xrightarrow{2V}$

5.2.9 Mise en série de source de courant

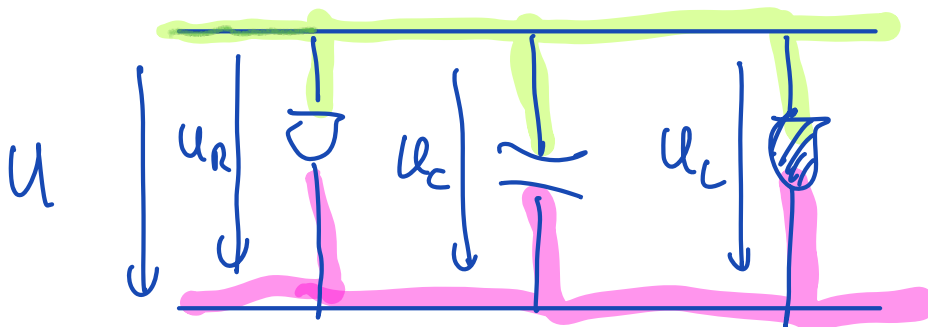


impossible

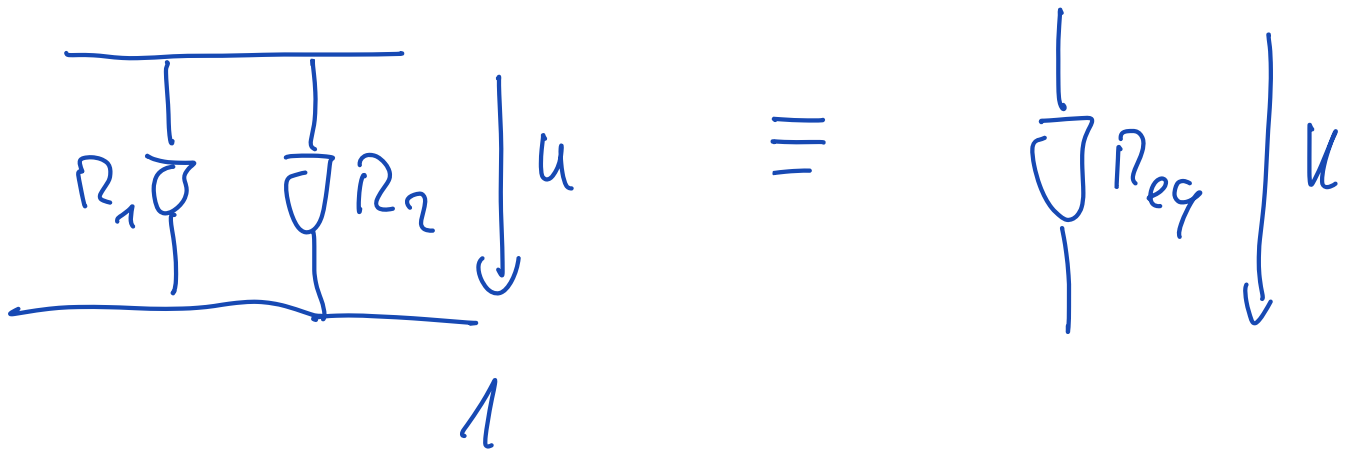
$\Rightarrow$ Impossible sauf si toutes les sources ont le même courant

5.3.2 Mise en // des R :

Définition : Toutes les bornes des éléments sont au même potentiel



$$U_R = U_C = U_L$$

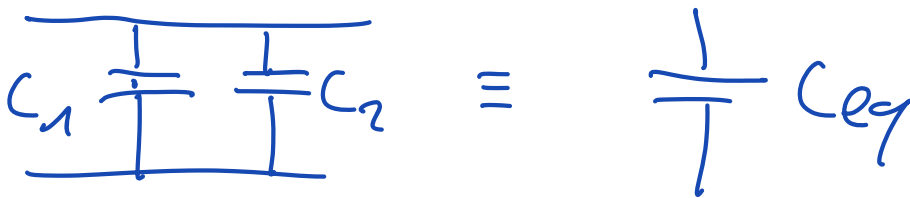


$$R_{eq} = \frac{1}{\sum_{k=1}^m \frac{1}{R_k}}$$

$$m = \text{nb de } R$$

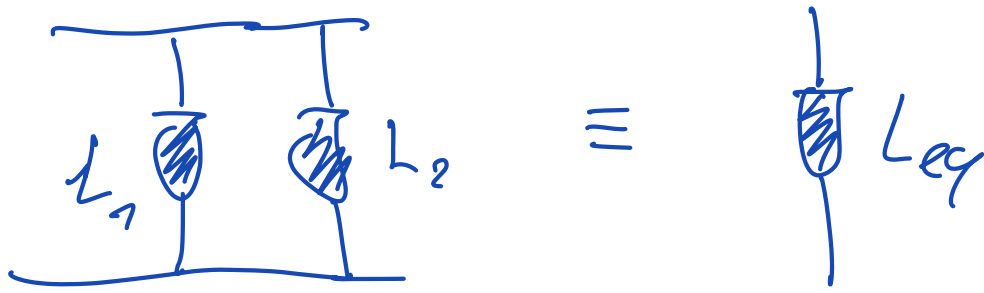
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

5.3.5 Mise en // des C :



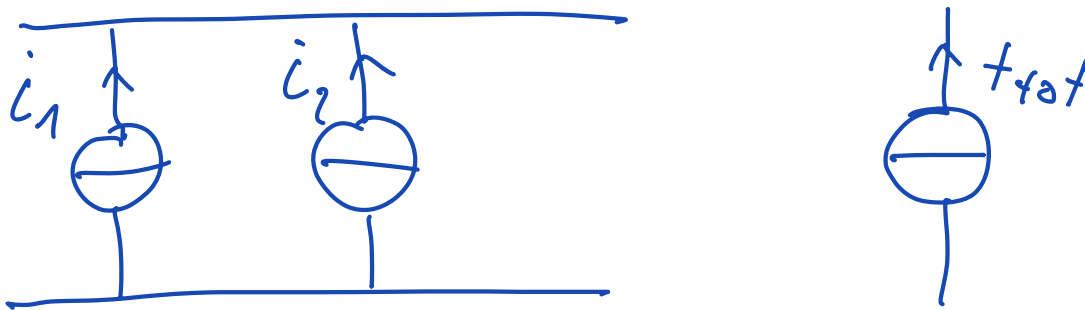
$$// \quad C_{eq} = \sum_{k=1}^m C_k \quad m = \text{nb de } C$$

5.3.6 Mise en // des L



$$// L_{eq} = \frac{1}{\sum_{k=1}^m \frac{1}{L_k}} \quad m = \text{nb de } L$$

5.3.7 Mise en // des sources de courant

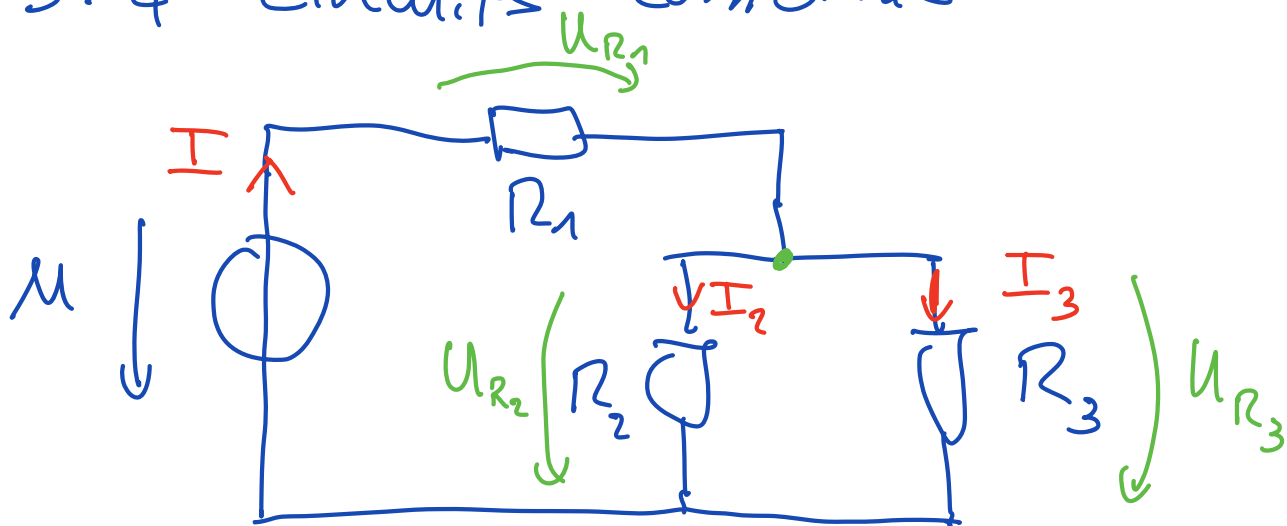


$$i_{tot} = \sum_{k=1}^m i_k$$

Mise en // de sources de tension

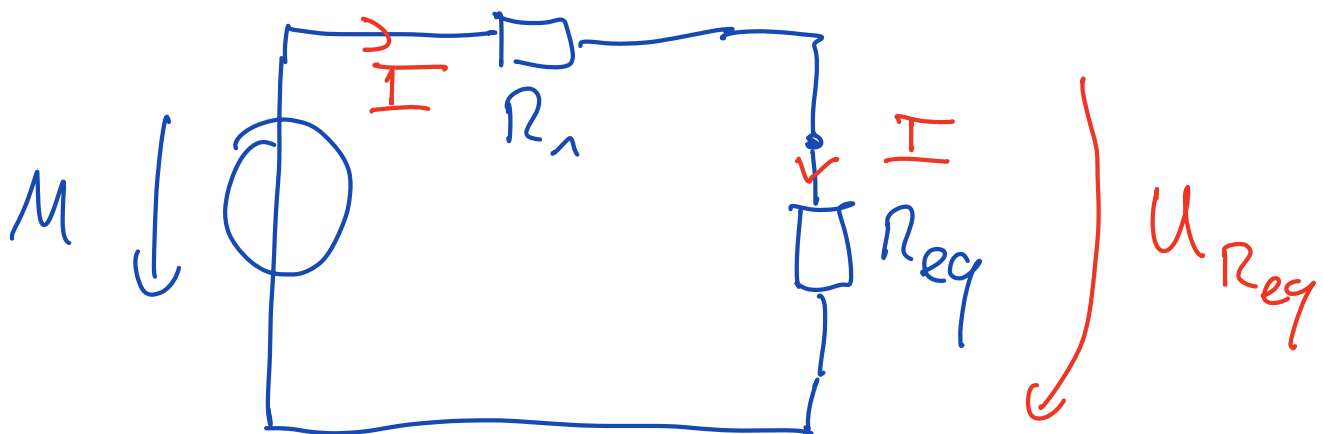
est impossible sauf si toutes les tensions ont la même valeur

5.4 Circuits combinés :

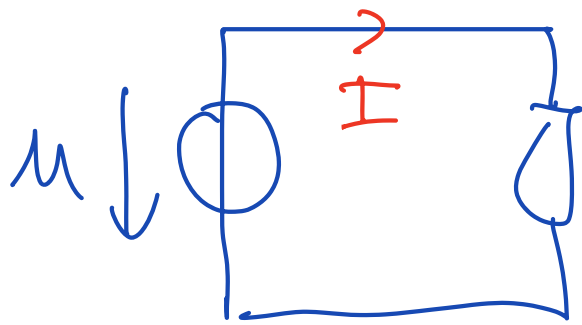


$$I = ? \quad I_2 = ? \quad I_3 = ?$$

$$R_2 \parallel R_3$$



$$R_{eq} = \frac{R_2 \cdot R_3}{R_2 + R_3}$$



$$R_{tot} = R_1 + R_{eq}$$

$$= R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}$$

$$U = R_{tot} \cdot I$$

$$I = \frac{U}{R_{tot}}$$

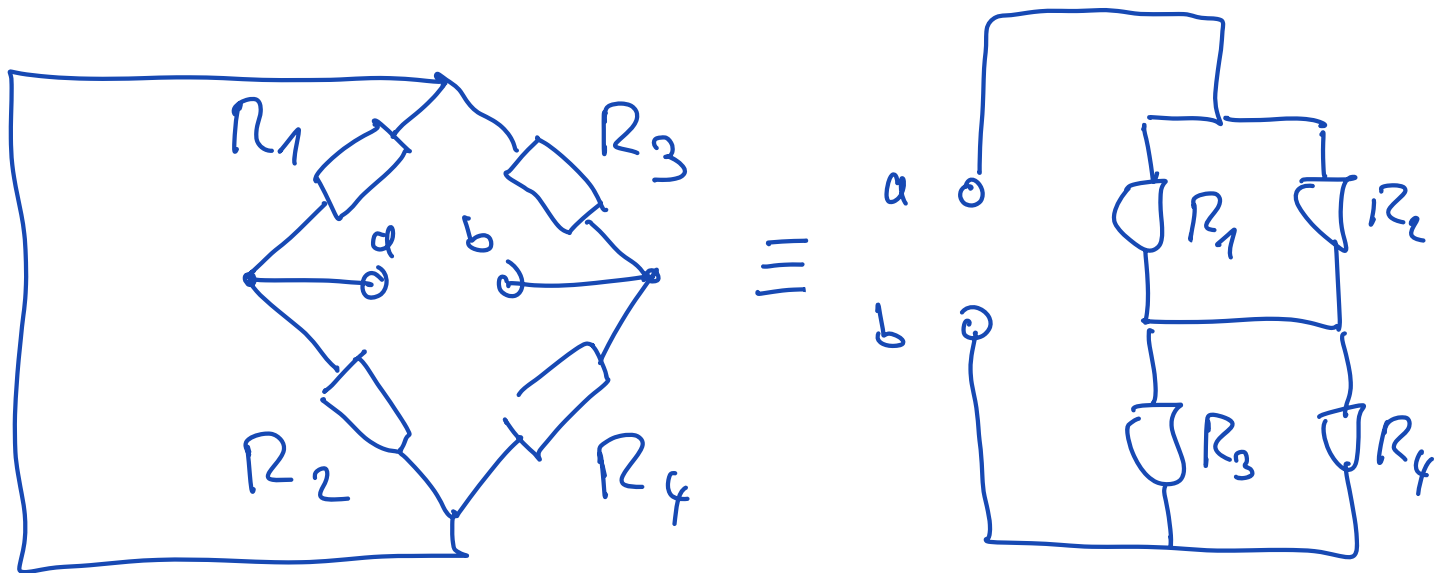
$$U_{R_2} = U_{R_3} = U_{R_{eq}} = R_{eq} \cdot I$$

$$= R_{eq} \cdot \frac{U}{R_{tot}}$$

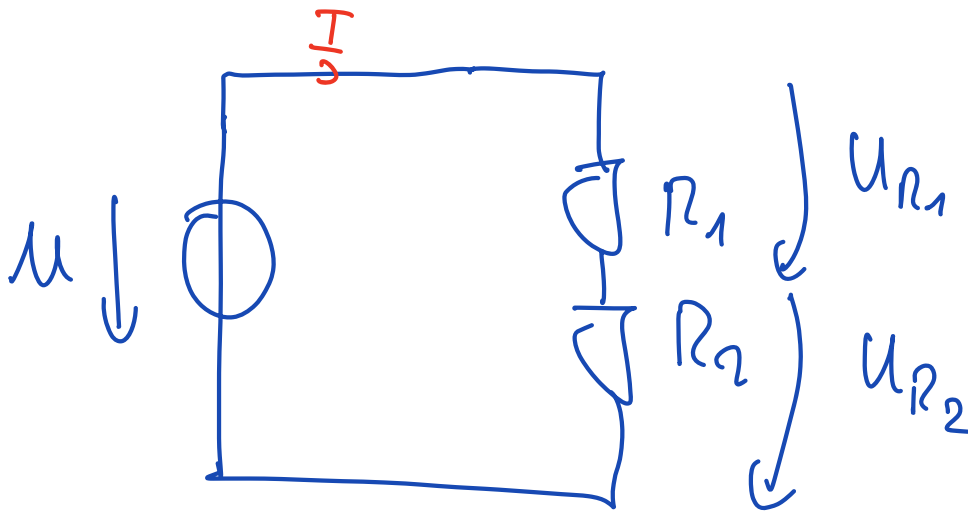
$$I_2 = \frac{U_{R_2}}{R_2} = \frac{U_{R_{eq}}}{R_2}$$

$$I_3 = \frac{U_{R_3}}{R_3} = \frac{U_{R_{eq}}}{R_3}$$

5.4.3 Exemple :



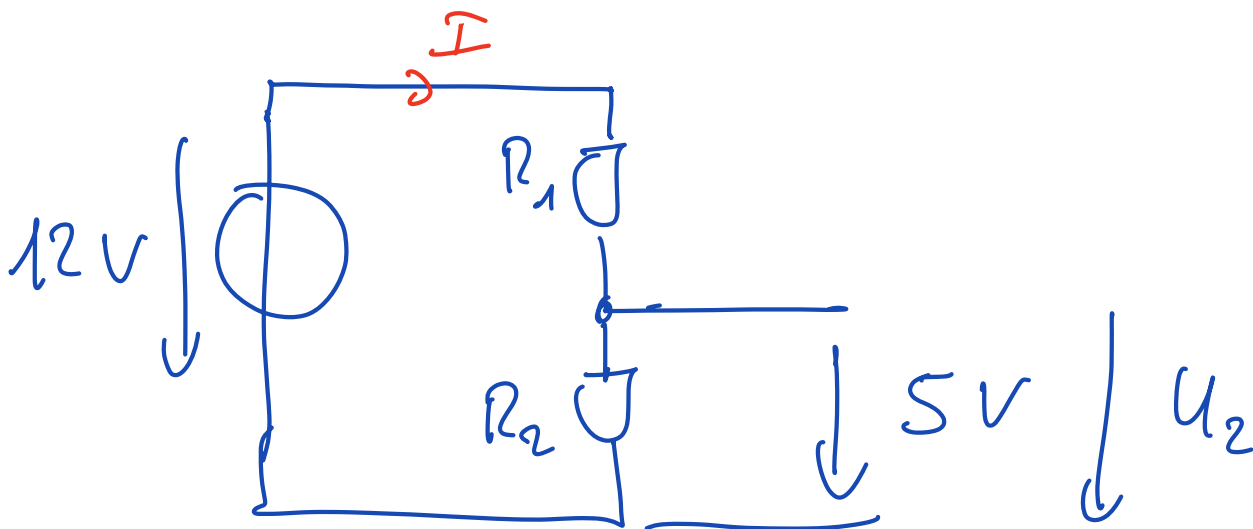
5.5.1 Diviseur de tension :



$$U = U_{R_1} + U_{R_2}$$
$$= (R_1 + R_2) I$$

$$I = \frac{U}{R_1 + R_2}$$

$$\underline{\underline{U_{R_2} = R_2 \cdot I = \frac{R_2}{R_1 + R_2} \cdot U}}$$



$$U_2 = \frac{R_2}{R_1 + R_2} \cdot U$$

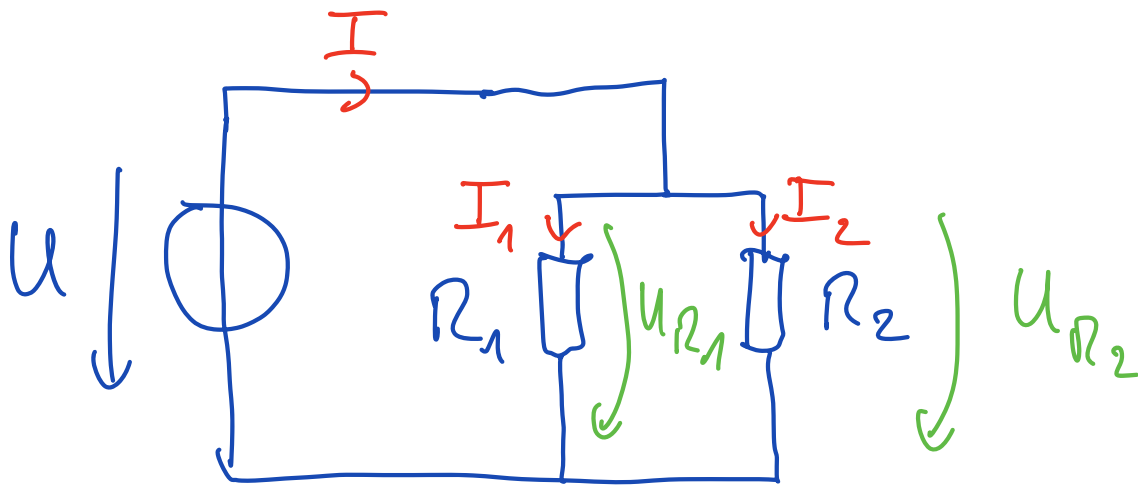
$$5 = \frac{R_2}{R_1 + R_2} \cdot 12$$

$$5R_1 = 7R_2$$

$$R_1 = 100 \text{ k}\Omega$$

$$R_2 = 71,5 \text{ k}\Omega$$

5.5.4 Diviseur de courant :



$$U = U_{R1} = U_{R2}$$

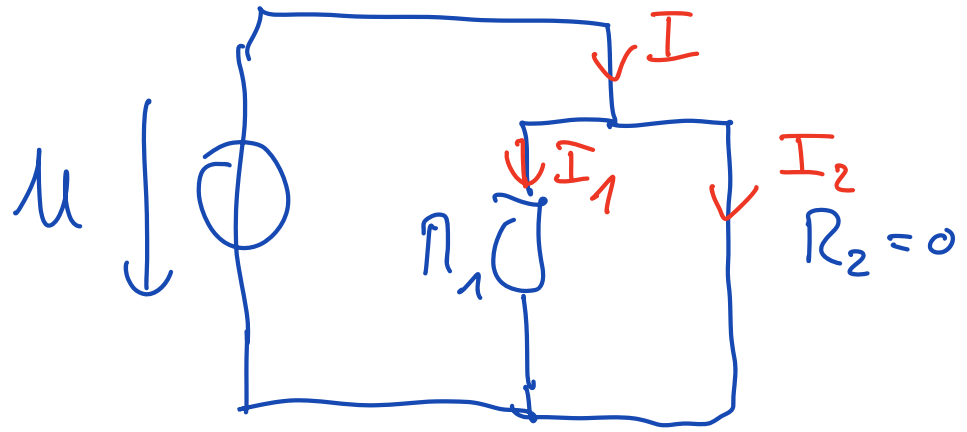
$$R_{eq} = \frac{R_1 \cdot R_2}{R_1 + R_2} \quad I = \frac{U}{R_{eq}}$$

$$U_{R2} = R_2 \cdot I_2 = U = \frac{R_1 \cdot R_2}{R_1 + R_2} \cdot I$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I$$

$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I$$

$S_i :$



$$I_2 = \frac{R_1}{R_1 + \cancel{R_2}} \cdot I = I$$

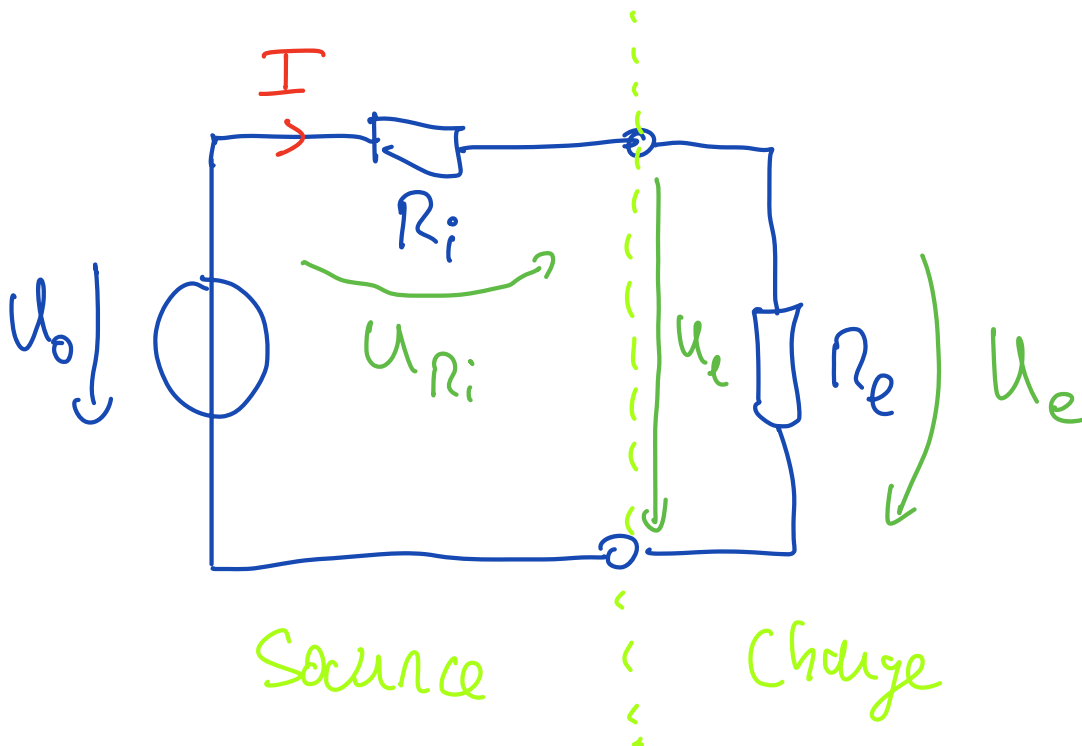
$$I_1 = 0$$

5.6 Méthodes de résolution:

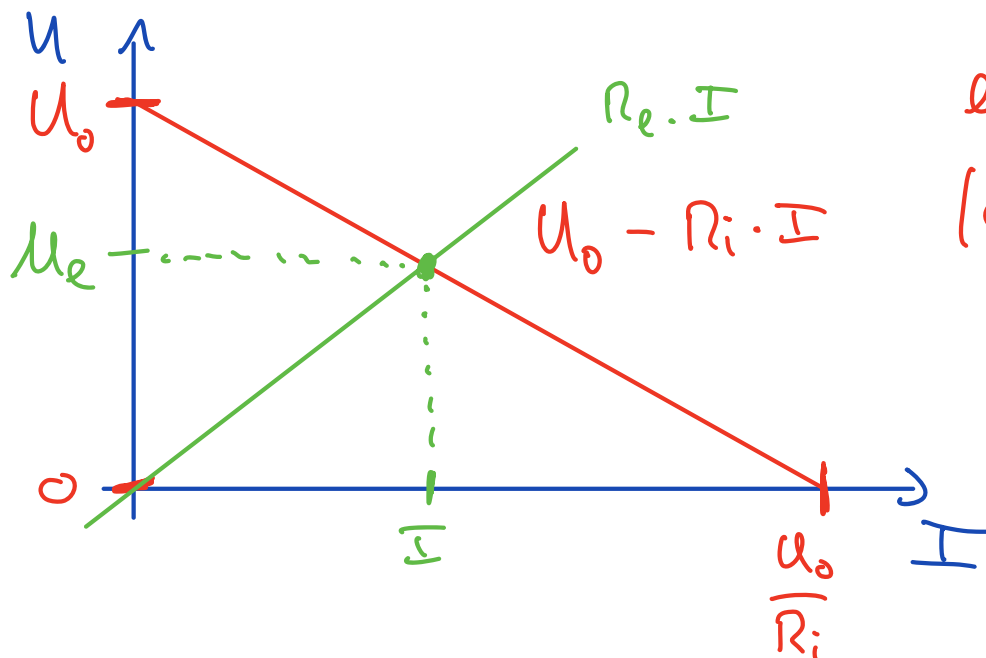
- Redessiner le schéma
- Définir toutes les grandeurs
 $U, I, \dots \rightarrow$ indice
- Définir le sens des flèches
- Réduire le schéma, série ou //

• Analyse !

5.6.2 Source de tension réelle :



$$U_e = U_0 - U_{R_i} = U_0 - R_i I$$



en court-circuit

$$(cc) : U_e = 0$$

$$I_{cc} = \frac{U_0}{R_i}$$

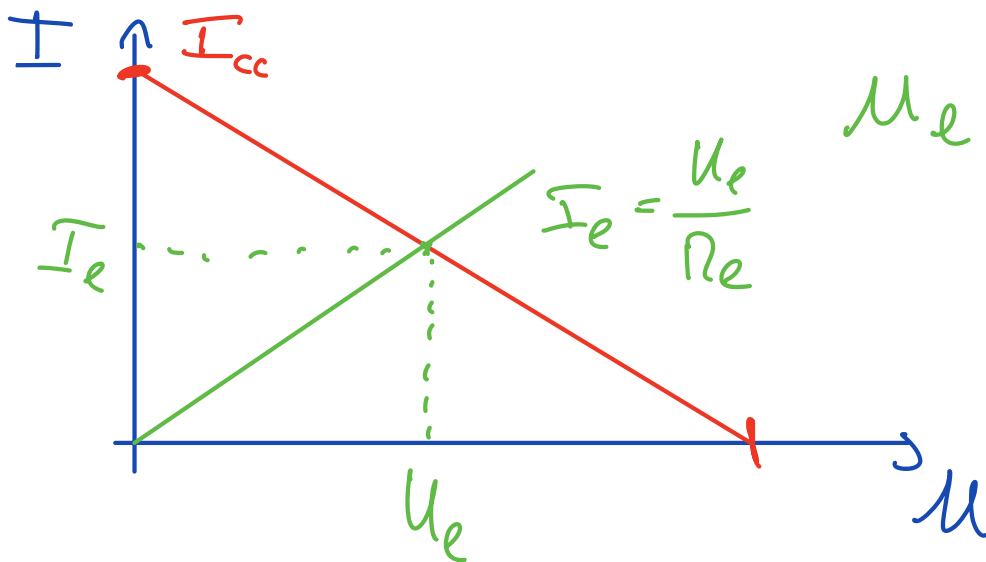
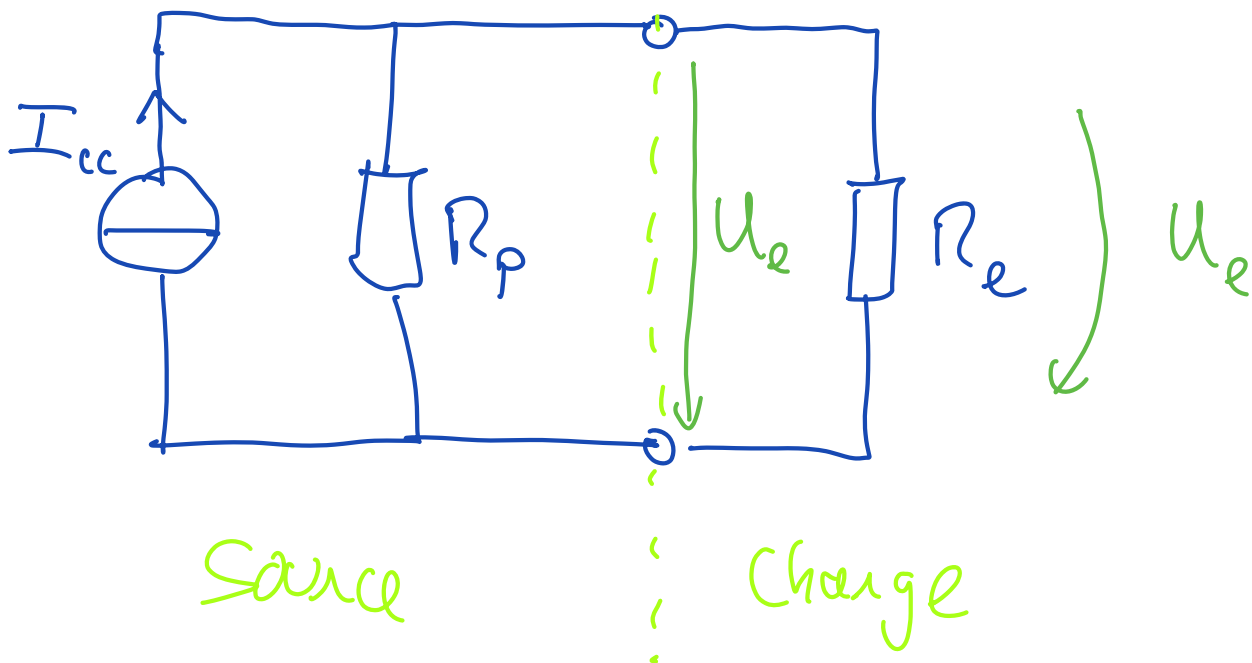
eq de la charge

$$U_e = R_e \cdot I$$

$$U_e = U_0 \cdot \frac{R_e}{R_e + R_i}$$

$$I_e = \frac{U_0}{R_i + R_e}$$

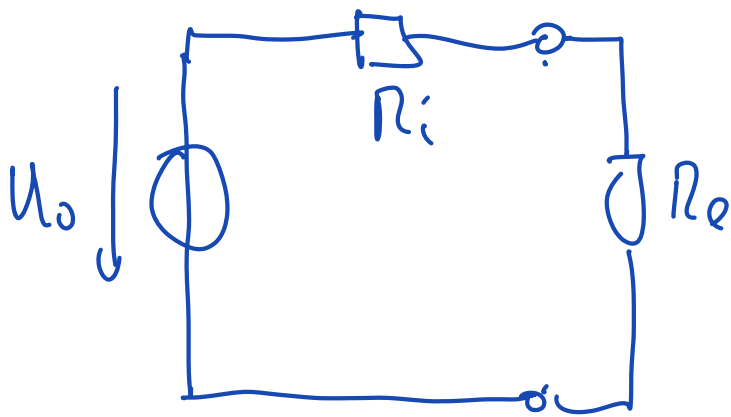
Source de courant réelle :



$$U_e = R_e \cdot I_e$$

$$I_e = \frac{U_e}{R_e}$$

5.6.3 Equivalence des Sources de tension et courant réelles



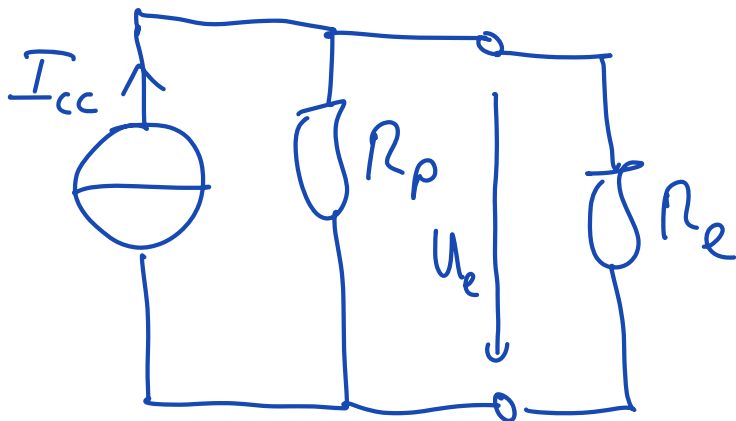
court-circuit
 $R_e = 0$

$$U_{e_{cc}} = 0$$

$$I_{e_{cc}} = \frac{U_0}{R_i}$$

circuit ouvert
 $R_e \rightarrow \infty$

$$U_{e_0} = U_0$$



$$I_{e_{cc}} = I_{cc}$$

$$U_{e_{cc}} = 0$$

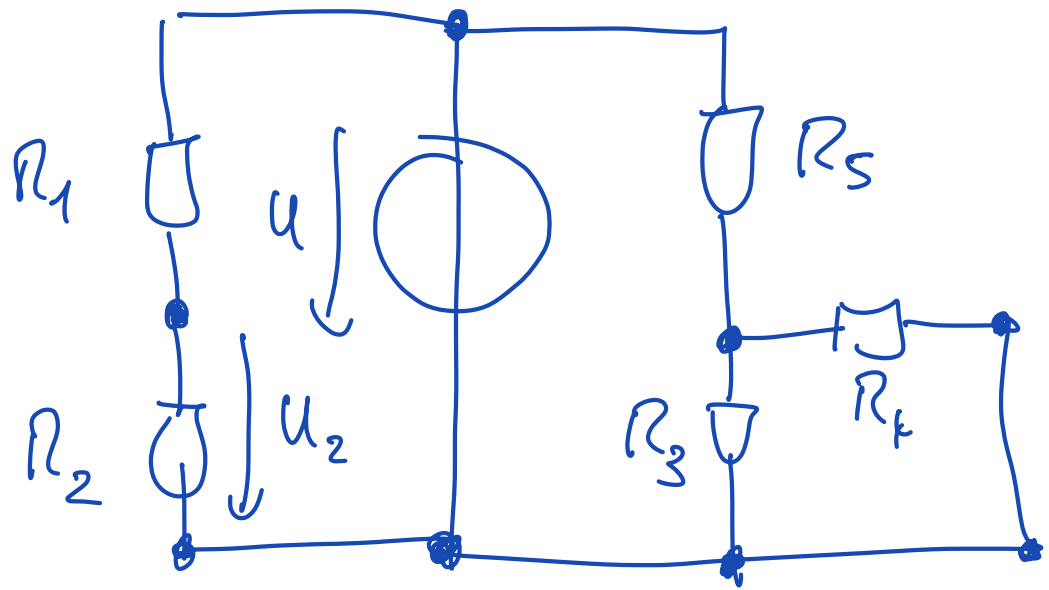
$$U_{e_0} = R_p \cdot I_{cc}$$

On pose : $U_{e_0} = R_p \cdot I_{cc} = U_0$

$$I_{e_{cc}} = \frac{U_0}{R_i} = I_{cc}$$

$$R_p = \frac{U_0}{I_{cc}} = \frac{U_0}{U_0/R_i} = R_i$$

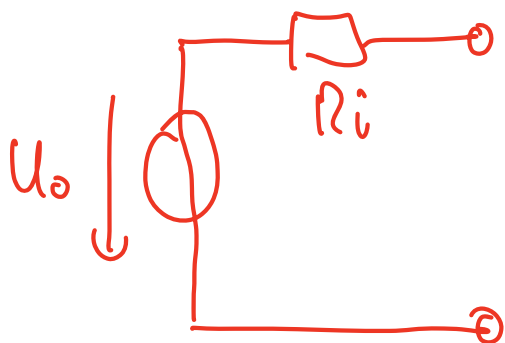
Quiz :



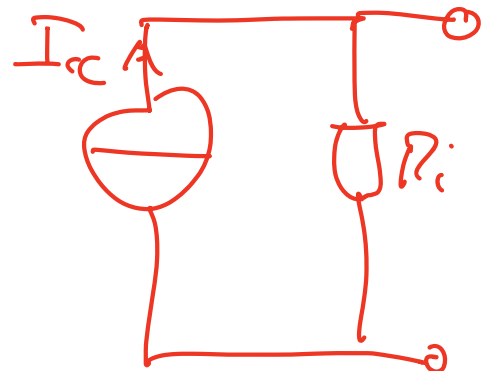
$$U = 12 \text{ V}$$

$$U_2 = U \cdot \frac{R_2}{R_1 + R_2} = 5 \text{ V}$$

En résumé :

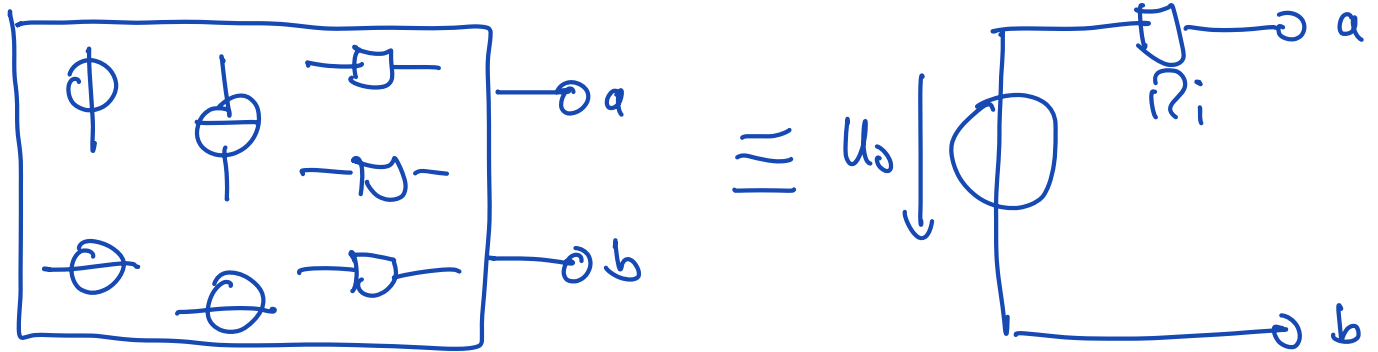


$\equiv$



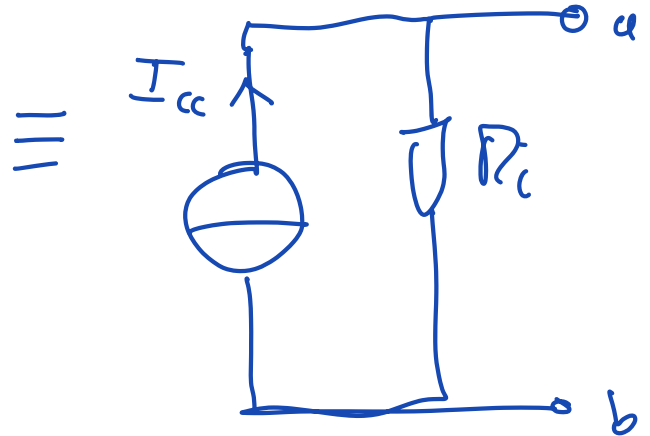
$$I_{cc} = \frac{U_0}{R_i}$$

5.7 Théorèmes de Thévenin et Norton:



$U_0 = \text{Tension à vide}$

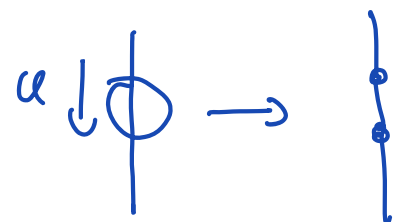
$$U_0 = U_{ab} \left| \begin{array}{l} (\text{à vide}) \\ I_{ab} = 0 \end{array} \right.$$

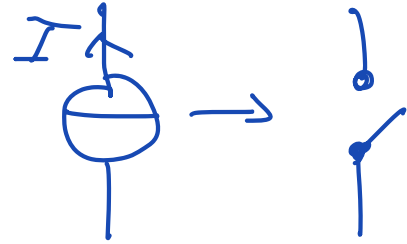


$$I_{cc} = I_{ab} \left| \begin{array}{l} (\text{en court-circuit}) \\ U_{ab} = 0 \end{array} \right.$$

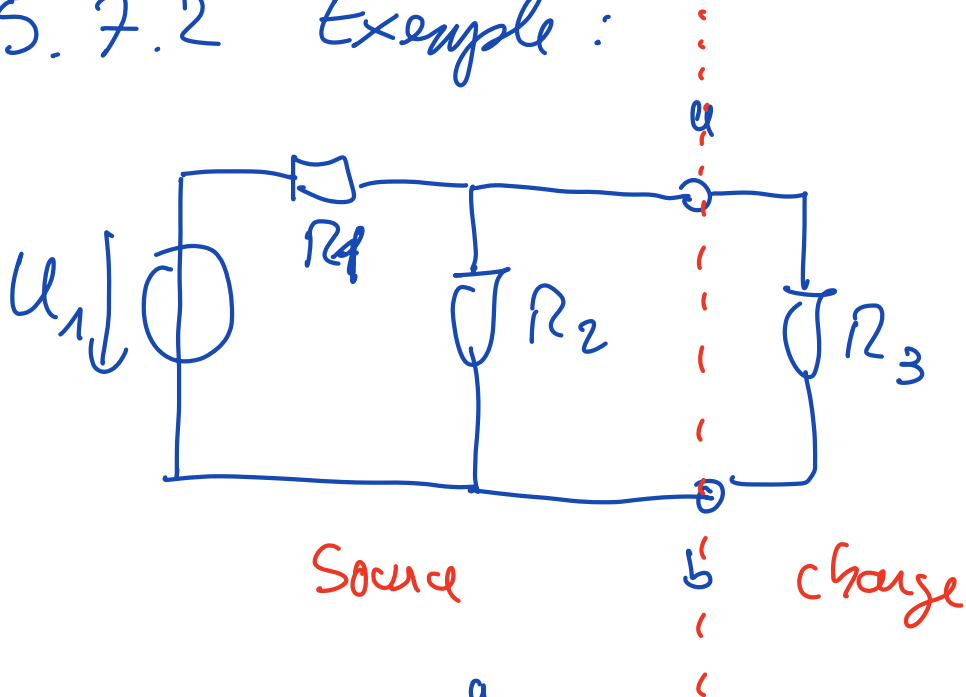
$$R_i = \frac{U_0}{I_{cc}} = R_{ab} \left| \begin{array}{l} U_j = 0 \\ I_j = 0 \end{array} \right.$$

Annuler une source $\Rightarrow$

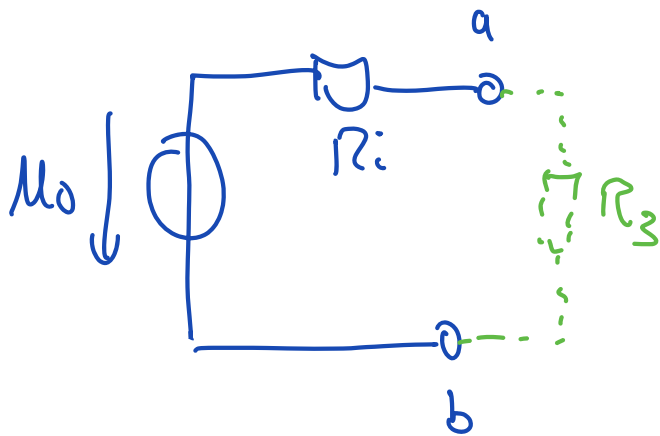




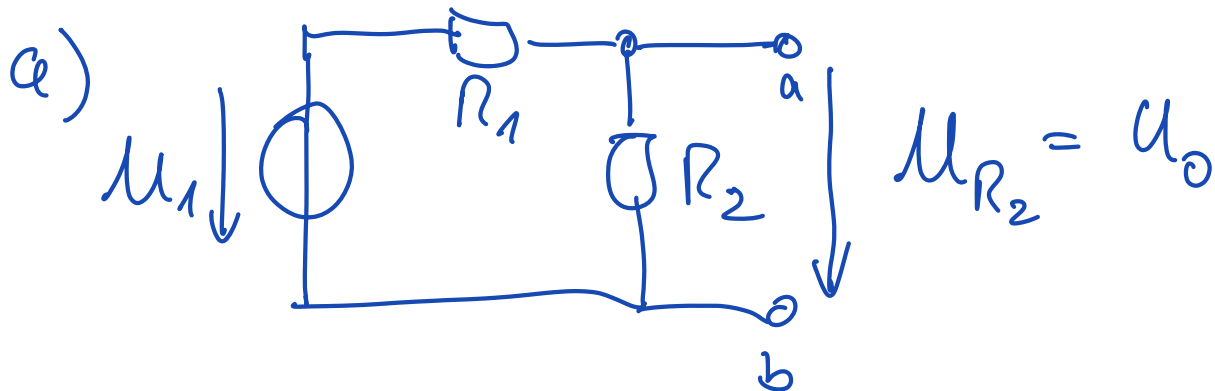
5.7.2 Example :



$$U_3 = f(R_3)$$

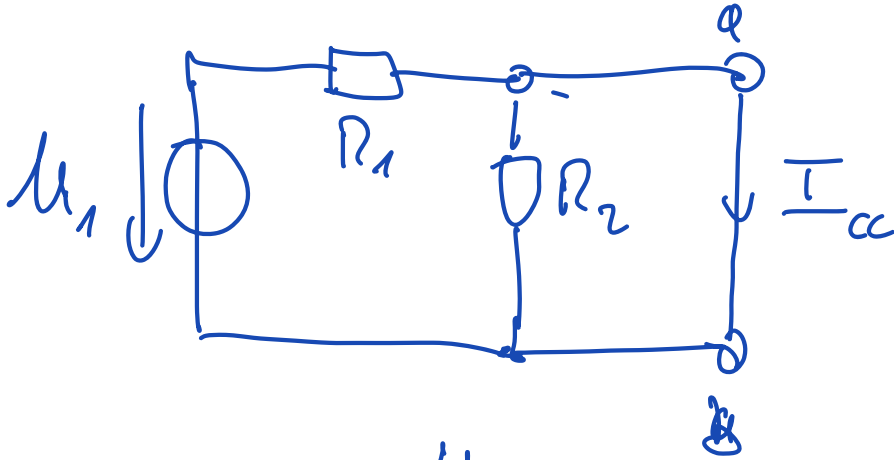


U_0 ? Tension à vide entre a et b ?



$$U_{R_2} = U_0 = U_1 \cdot \frac{R_2}{R_1 + R_2}$$

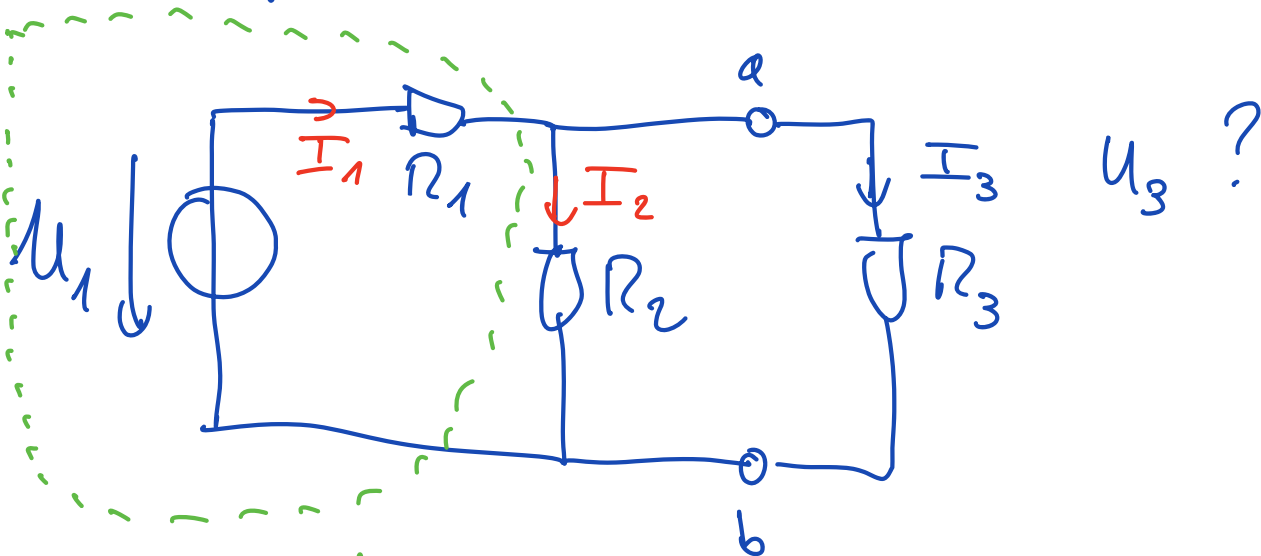
b) I_{cc} = courant de court circuit



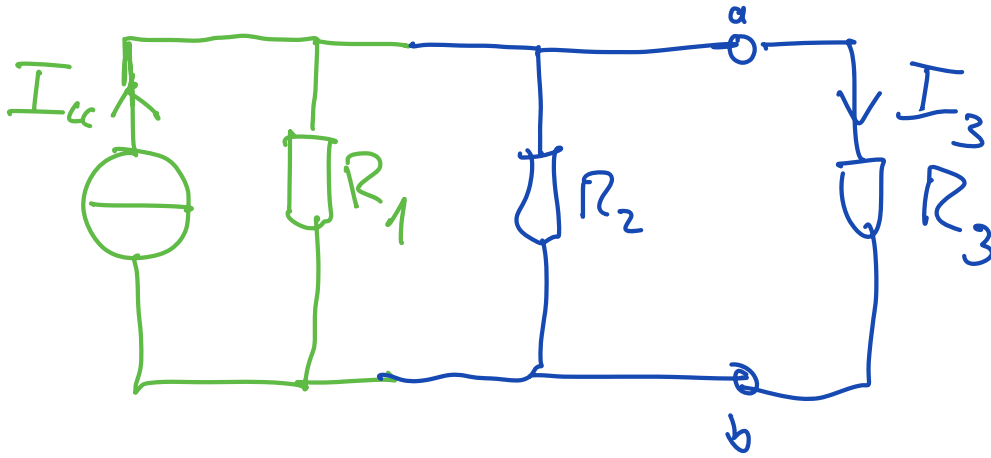
$$I_{cc} = \frac{U_1}{R_1}$$

c) $R_i = \frac{U_o}{I_{cc}} = \frac{R_1 \cdot R_2}{R_1 + R_2}$

Autre possibilité :

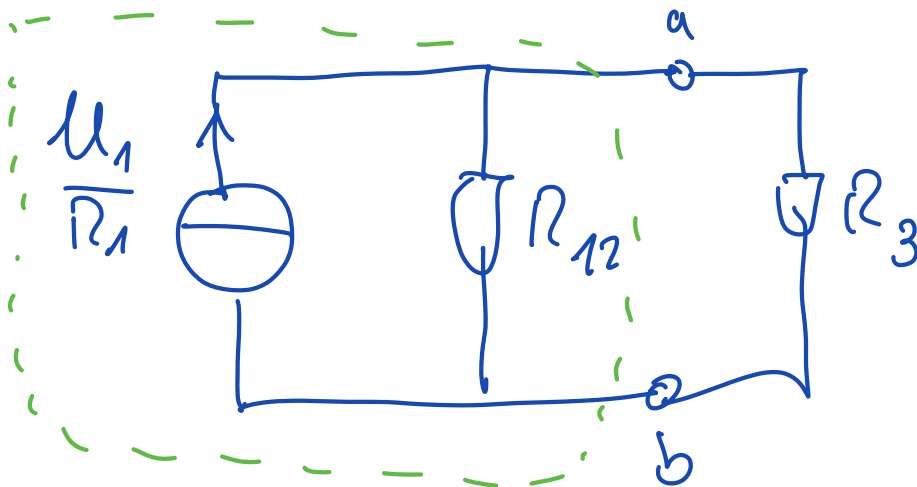


Source de tension
réelle

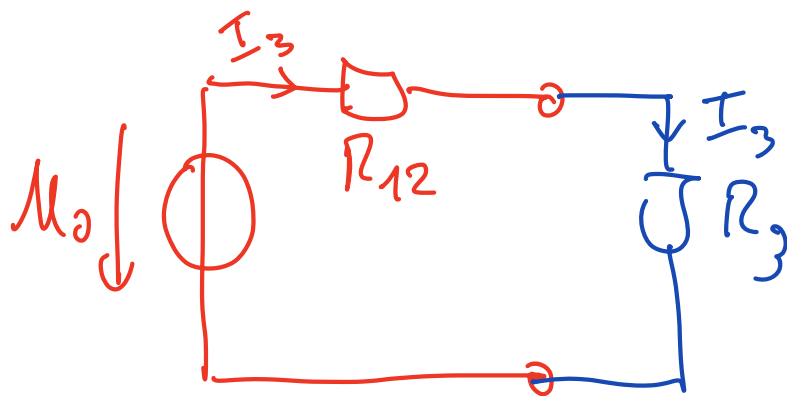


$$I_{cc} = \frac{U_1}{R_1}$$

$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



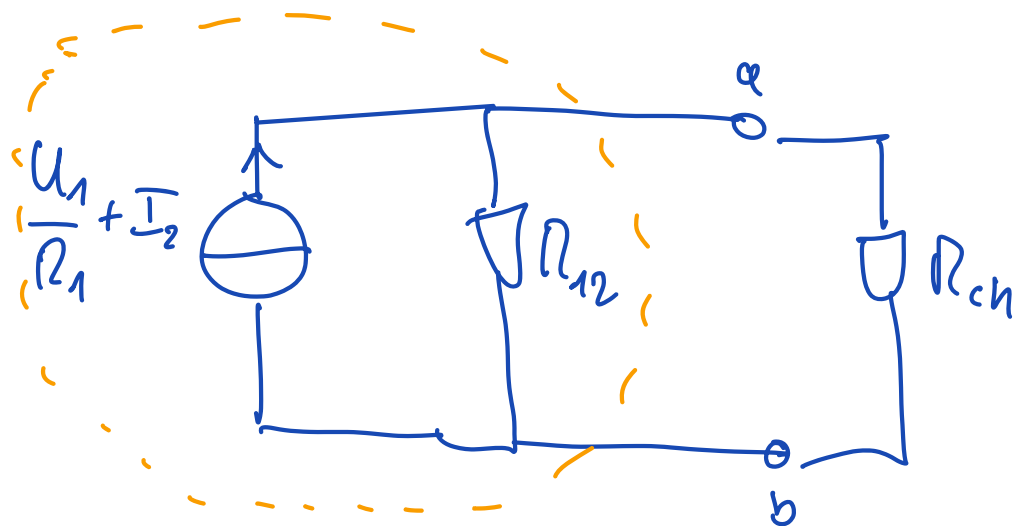
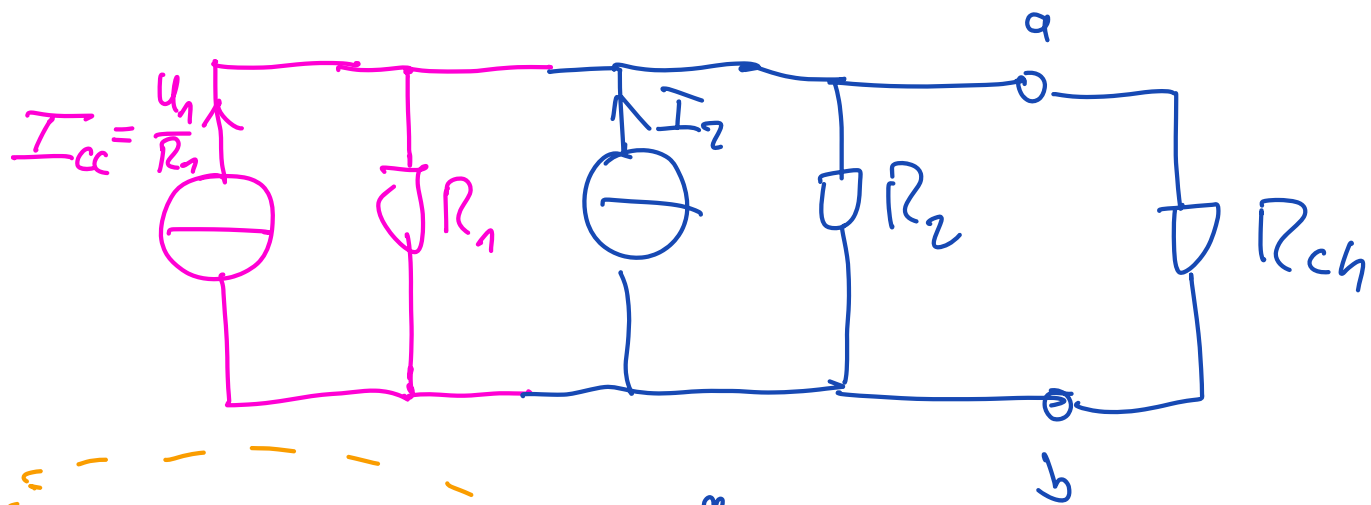
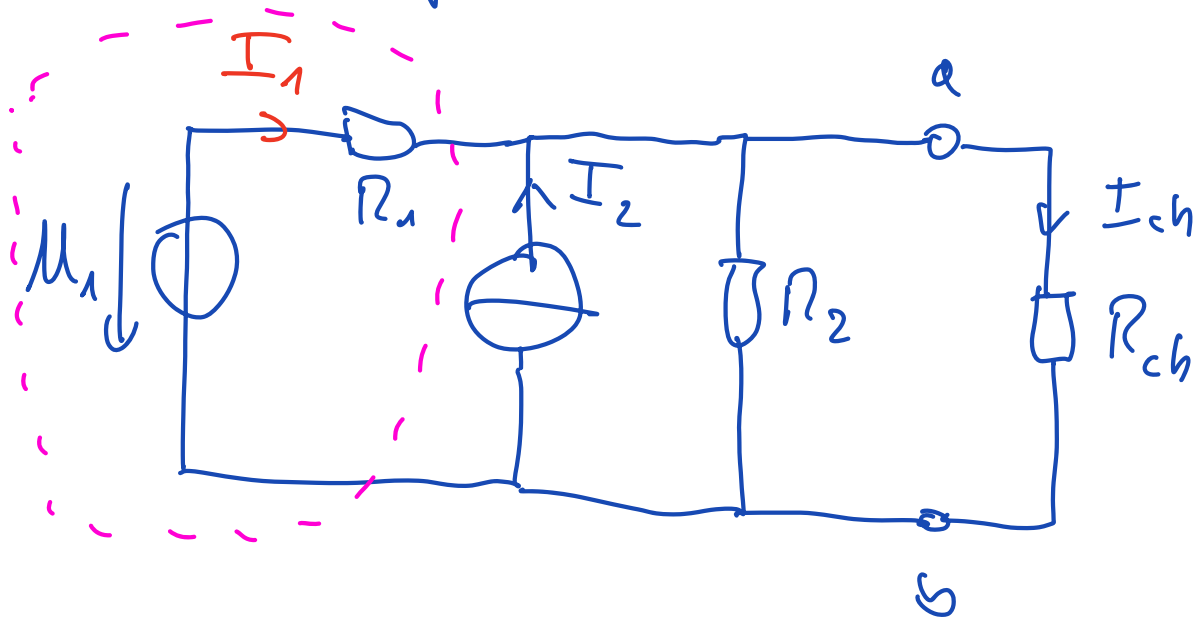
Source de courant réelle



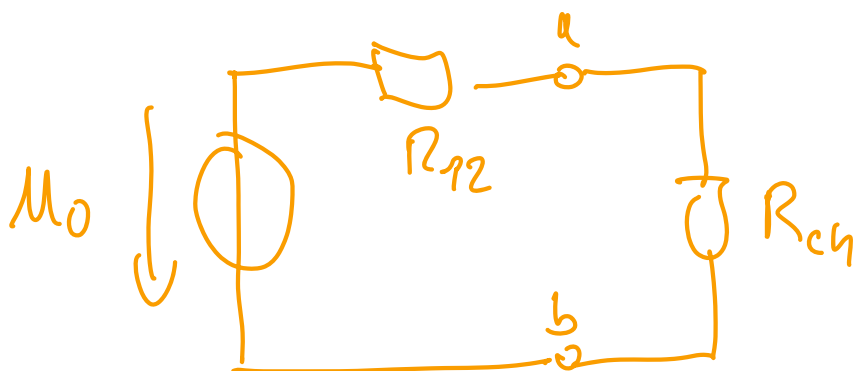
$$U_0 = R_{12} \cdot \frac{U_1}{R_1}$$

$$I_3 = \frac{U_0}{R_{12} + R_3}$$

Another example:



$$R_{12} = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

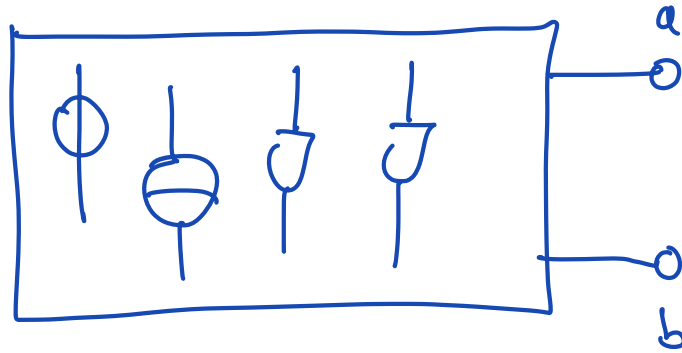


$$U_0 = R_{12} \left(\frac{U_1}{R_1} + I_2 \right)$$

$$I_{ch} = \frac{U_0}{R_{12} + R_{ch}}$$

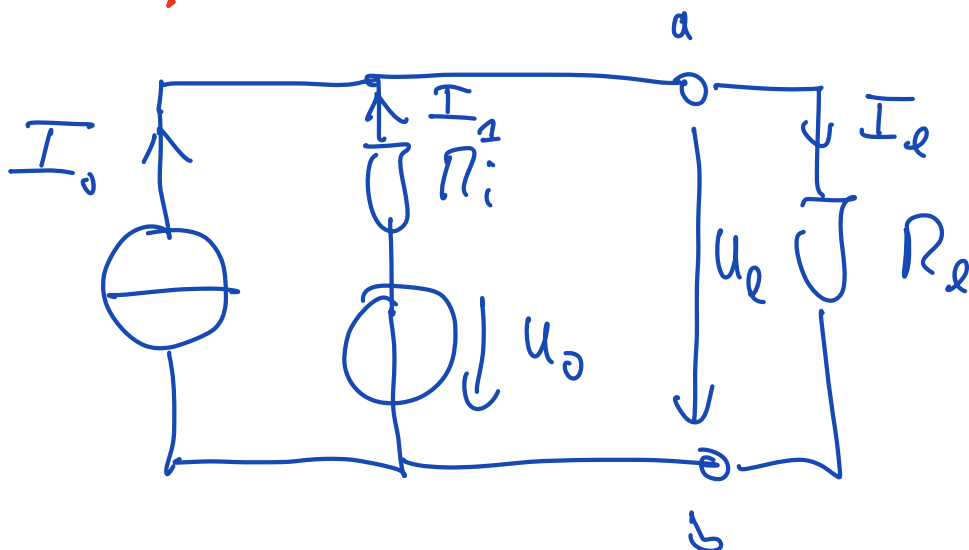
5.8 Principe de Superposition

Définition :



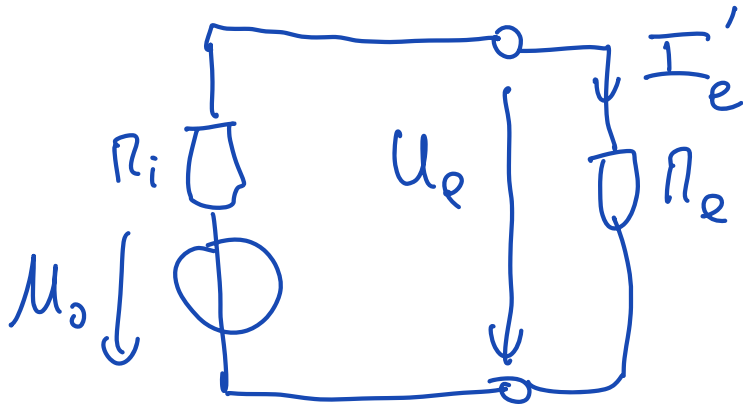
L'action résultante est la somme algébrique des actions séparées de chaque source, les autres étant annulées

Le système doit être linéaire !!



$I_e, u_e ?$

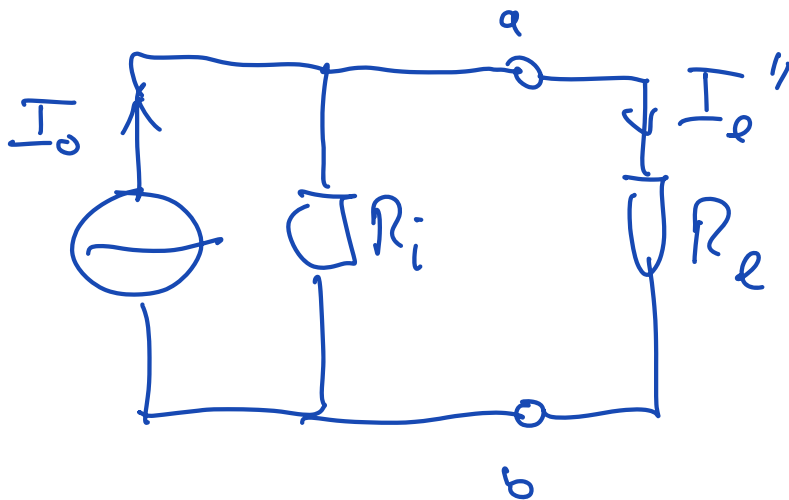
1) On annule la source de courant :



$$I_e' = \frac{U_0}{R_i + R_e}$$

$$U_e' = U_0 \cdot \frac{R_e}{R_e + R_i}$$

2) On annule la source de tension :



$$I_e'' = I_0 \cdot \frac{R_i}{R_i + R_e}$$

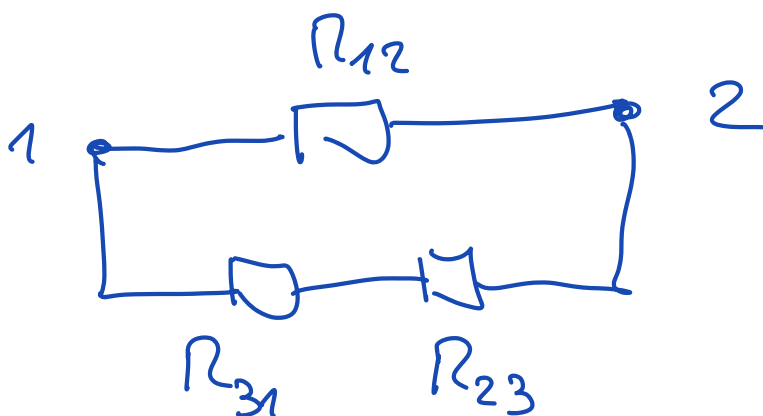
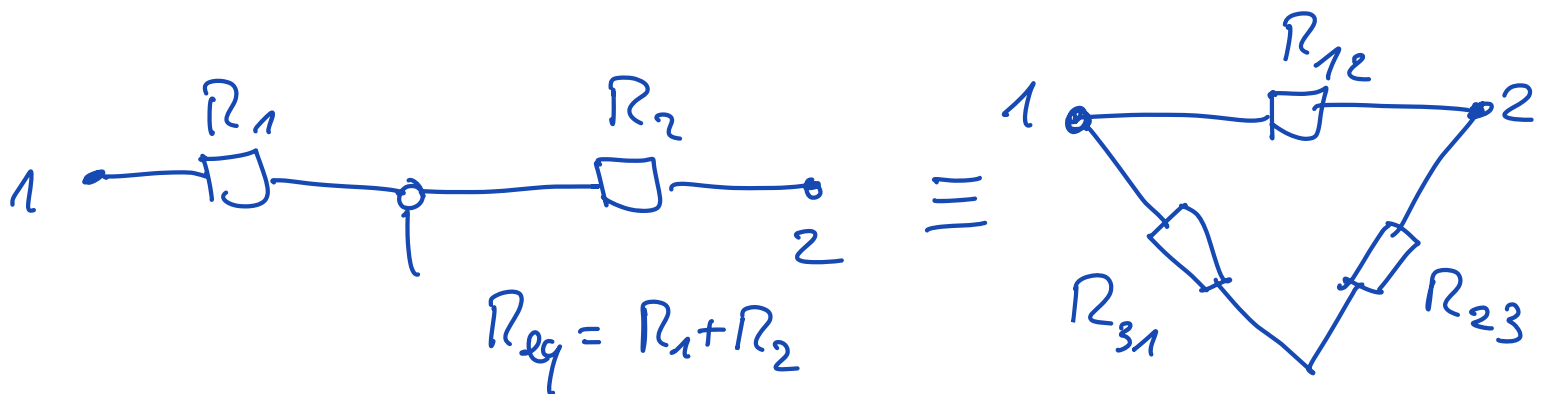
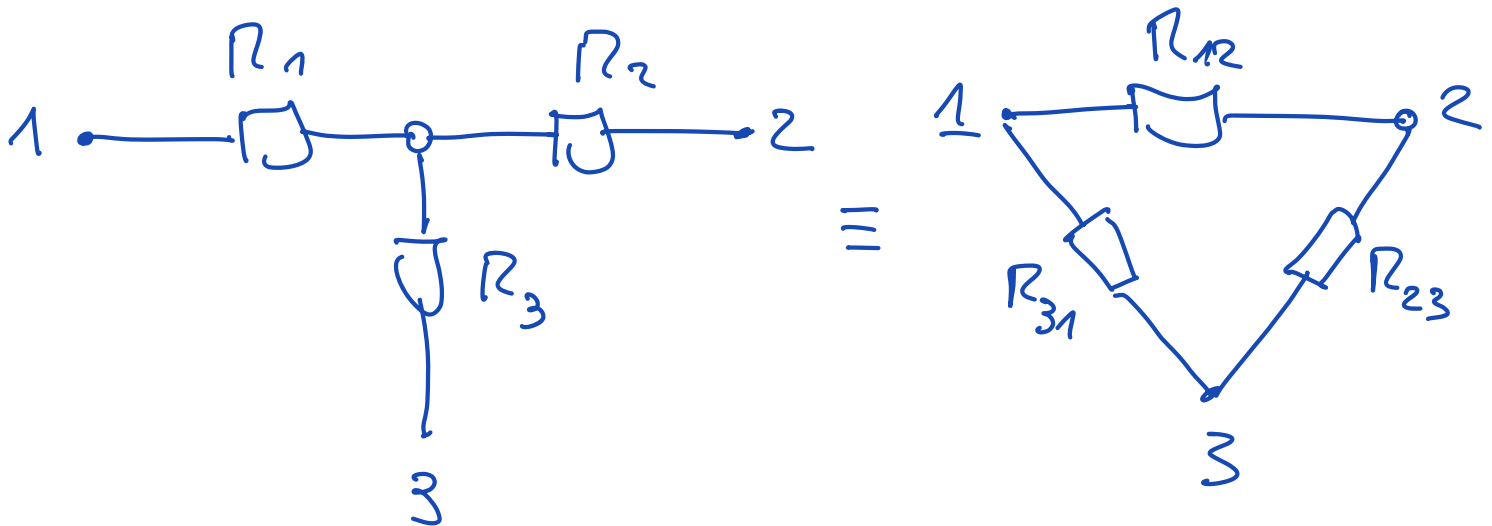
$$U_e'' = R_e \cdot I_e'' = I_0 \cdot \frac{R_e \cdot R_i}{R_e + R_i}$$

$$I_e = I_e' + I_e''$$

$$M_e = M_e' + M_e''$$

5.9 Transformation Étoile - Triangle

il s'agit d'un tripole :



$$R_{eq} = \frac{R_{12} \cdot (R_{31} + R_{23})}{R_{12} + R_{31} + R_{23}}$$

$$R_1 = \frac{R_{31} \cdot R_{12}}{R_{12} + R_{23} + R_{31}}$$

$$R_{12} = R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}$$

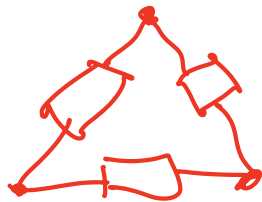
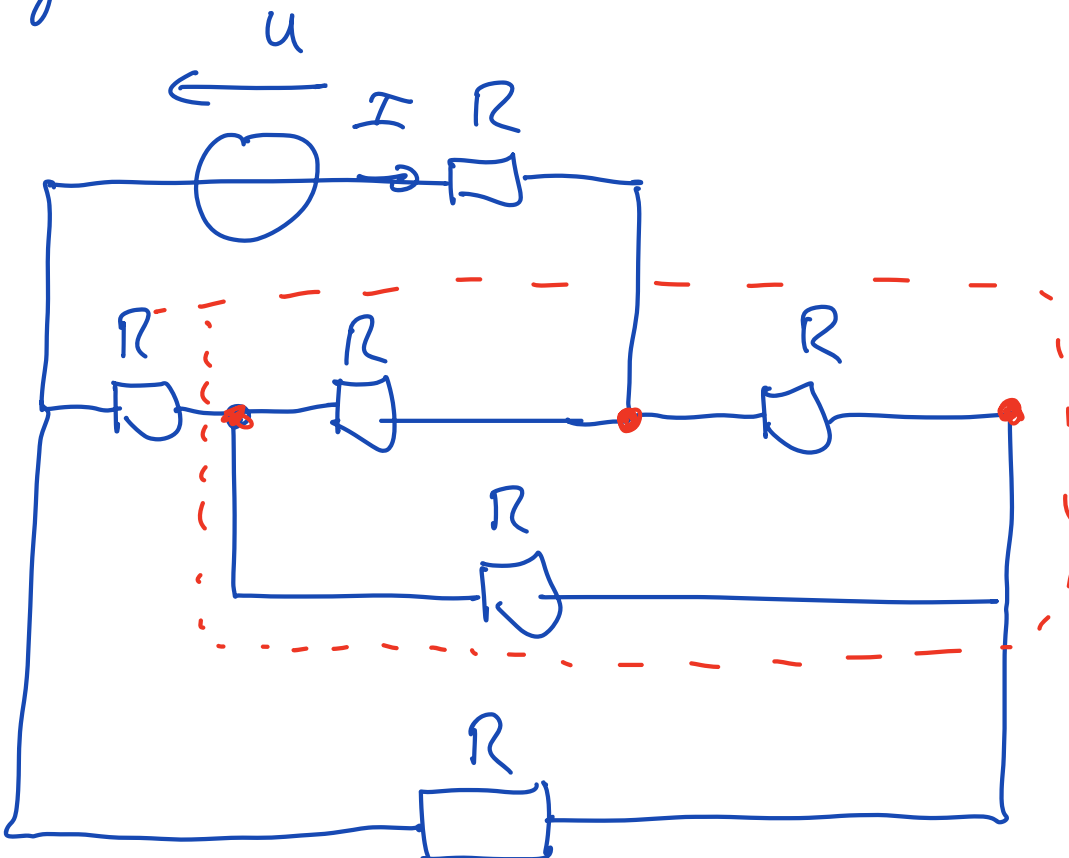
$$R_2 = \frac{R_{12} \cdot R_{23}}{R_{12} + R_{23} + R_{31}}$$

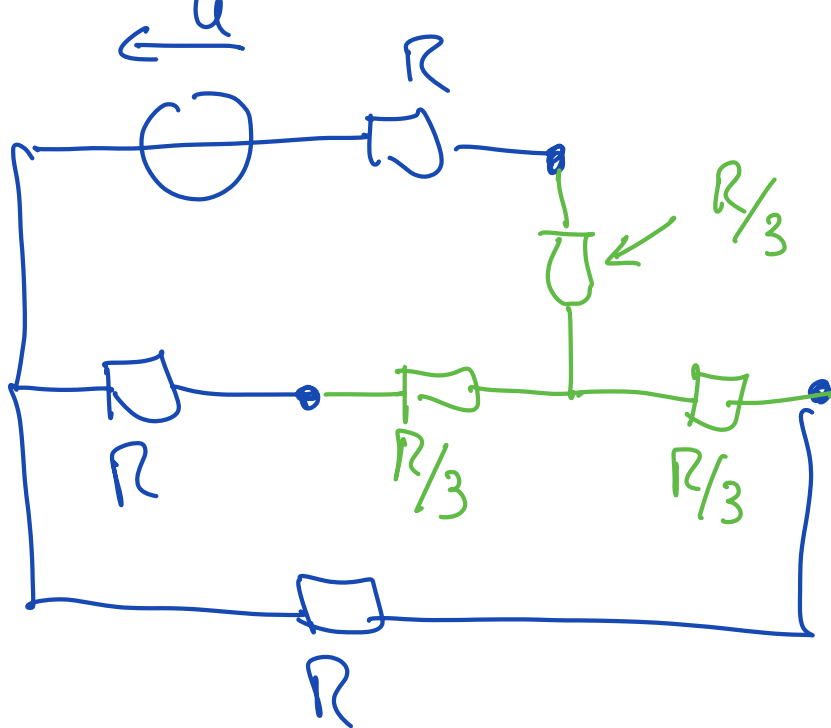
$$R_{23} = R_2 + R_3 + \frac{R_2 \cdot R_3}{R_1}$$

$$R_3 = \frac{R_{23} \cdot R_{31}}{R_{12} + R_{23} + R_{31}}$$

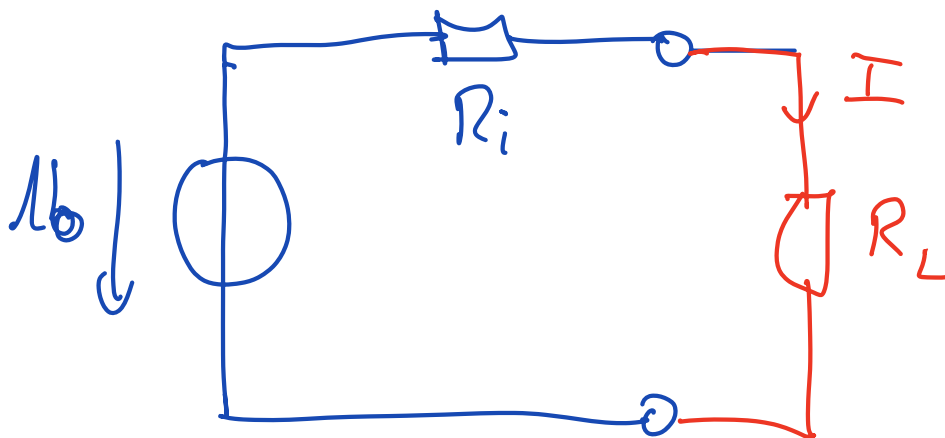
$$R_{31} = R_3 + R_1 + \frac{R_3 \cdot R_1}{R_2}$$

Example :





5.11 Puissance Maximum transmise
par un dipôle :



Puissance
transmise à
 $R_L \rightarrow P_{\text{ox}}$

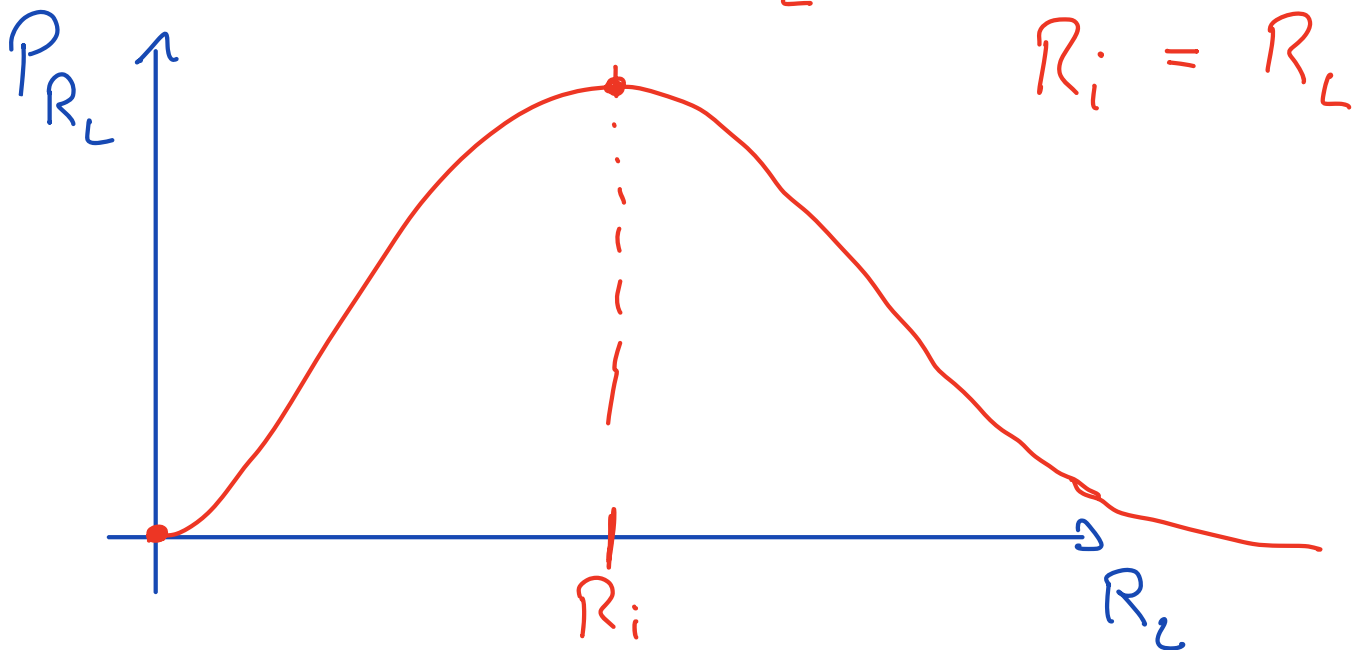
$$P_{R_L} = R_L \cdot I^2 = U_{R_L} \cdot I$$

$$I = \frac{U_0}{R_i + R_L}$$

$$P_{R_L} = R_L \frac{U_0^2}{(R_i + R_L)^2}$$

Qui doit valoir R_L par rapport à R_i pour P_{R_L} max

Max : $\rightarrow \frac{dP_{R_L}}{dR_L} = 0$



$\rightarrow$ d'adaptation de puissance

$$\eta = \frac{P_{\text{utile}}}{P_{\text{consommée}}} = \frac{R_L \cdot \cancel{I^2}}{(R_L + R_i) \cancel{I^2}}$$

$$S_i \quad R_i = R_L$$

$$\Rightarrow \eta = 0,5$$